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**Department: Mathematics**

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## **Assignment-2**

**Subject: Mathematics-II**

**(BT-202)**

**(Common to All Branches)**

**Topic: Ordinary Differential Equation-II**

1. Solve the equation  $x^2 \frac{d^2 y}{dx^2} - (2x-1) \frac{dy}{dx} + (x-1)y = e^x$  given that  $e^x$  is one integral.
2. Solve the equation  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$ , given that  $x + \frac{1}{x}$  is one integral.
3. Solve by removal of first derivative
  - (i)  $\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = \sec x \cdot e^x$
  - (ii)  $\frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \cdot \sin 2x$
4. Solve by change of independent variable
  - (i)  $\frac{d^2 y}{dx^2} + \tan x \frac{dy}{dx} - 2 \cos^2 x \cdot y = 2 \cos^4 x$
  - (ii)  $x \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 4x^3 y = 8x^3 \cdot \sin(x^2)$
5. Apply the method of variation of parameters to solve
  - (i)  $\frac{d^2 y}{dx^2} + 4y = 4 \tan 2x$
  - (ii)  $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$
  - (iii)  $\frac{d^2 y}{dx^2} + a^2 y = \operatorname{cosec} ax$

6. Solve in series of the following equation when  $x = 0$  is a ordinary point

(i)  $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

(ii)  $(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$

(iii)  $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$

7. Solve in series of the following equation when  $x = 0$  is a Regular singular point

(i)  $2x^2\frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$

(ii)  $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + (x^2 - n^2)y = 0$

8. Prove That

(i)  $P_n(-x) = (-1)^n P_n(x)$

(ii)  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

(iii)  $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{\sin x}{x} - \cos x \right)$

9. Express  $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$  in terms of Legendre Polynomials.

10. Show that:

(i)  $P_n(-1) = (-1)^n$

(ii)  $P'_n(1) = \frac{1}{2}n(n+1)$

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