LNCT GROUP OF COLLEGES

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Department: Mathematics

LNCT&S, Bhopal (M.P)

Assignment-5

Subject: Mathematics-II

(BT-202)

(Common to All Branches)

Topic: Vector Calculus

1. If $\vec{a} = t^2 i - t\,j + (2t+1)k$ and $\vec{b} = \geq (2t-3)i + j + t\,k$ Find $\frac{d}{dt}(\vec{a}..\vec{b})$ at $t = 1$.
2. Define gradient of scalar field. Find gradient of $\phi(x, y, z) = x^2 + y^2 - x$ at point $(1, 2, 5)$.
3. Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point (-1,-1, 2).
4. Show that the vector $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ is irrotational.
5. Find $div\,\vec{F}$ and $curl\,\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$
6. Prove that the vector $\vec{V} = 3y^4z^2\,i + 4x^3z^2\,j - 3x^2y^2\,k$ is Solenoidal.
7. Show that a vector field given by $\vec{A} = (x^2 + xy^2)i + (y + x^2y)j$ is irrotational. Find the scalar potential and find its scalar potential.
8. For a Solenoidal vector $\vec{F}$, Show that $curl.curl.curl.curl\,\vec{F} = \nabla^4\vec{F}$
9. Show that $div\,(grad\ r^m) = \nabla.\nabla r^m = m\,(m+1)r^{m-2}$, where $r^2 = x^2 + y^2 + z^2$
10. To Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $r^2 = x^2 + y^2 + z^2$.
11. Find the directional derivative of function of $\phi = xy + yz + zx$ in the direction of the vector $i + 2j + 2k$ at the point $(1, 1, 2)$. Also find the maximum value of directional derivative at the point.

12. Find the work done when a force $\vec{F} = (x^2 + y^2 + x)i - (2xy + y)j$ moves a particle in the xy-plane from (0, 0) to (1, 1) along the parabola $y^2 = x$. Is the work done different when the path is straight line $y = x$?

13. Evaluate: $\iint_S \vec{F}.\hat{n}\,.ds$, where $\vec{F} = 4x\,i - 2y^2 j + z^2 k$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

14. Write a statement of Gauss divergence theorem and Verify Divergence theorem for $\vec{F} = x^2 i + z\,j + yzk$ taken over the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0\ and\ z = 1$.

15. Write a statement of Stoke's Theorem and Verify Stoke's Theorem for the vector field $\vec{F} = (2x - y)i - yz^2 j - y^2 z\,k$ over the upper half of surface $x^2 + y^2 + z^2 = 1$ bounded by its projection on XY plane.

16. Using stoke's theorem, evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = (x^2 + y^2)i - 2xyj$ and c is rectangle bounded by $x = \pm a,\ y = 0\ and\ y = b$

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