LNCT GROUP OF COLLEGES

Name of Faculty: Dr. Sanjeet Kumar

Designation: Professor & Head

Department: Mathematics

LNCT&S, Bhopal (M.P)

Assignment-4

Subject: Mathematics-II

(BT-202)

(Common to All Branches)

Topic: Functions of Complex Variable

1. Find the imaginary part of the analytic function whose real part is $x^3-3xy^2-3x^2-3y^2$
2. Show that the function $u=2x-x^3+3xy^2$ is harmonic. Find its harmonic conjugate.
3. Show that the function $u=x^3-3xy^2$ is harmonic and Find the corresponding analytic function of this as the real part.
4. Using Cauchy Integral formula Prove that

   (i) $\int_c \frac{e^{2z}}{(z+1)^4}dz=\frac{8\pi e^{-2}i}{3}$ where c is the circle $|z|=3$

   (ii) $\int_c \frac{dz}{(z-a)}=2\pi i$ where c is the circle $|z-a|=r$
5. Evaluate the integral by Cauchy Integral formula.

   (i) $\int_c \frac{\cos\pi z}{(z-1)(z+1)}dz$, where c is the circle $|z|=3$.

   (ii) $\int_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$, where c is the circle $|z|=3$

   (iii) $\int_c \frac{(4-3z)}{z(z-1)(z-2)}dz$, where c is the circle $|z|=\frac{3}{2}$

6. Evaluate $\int_0^{2+i} \left(\bar{z}\right)^2 dz$,along,

(i) The real axis to2 and then vertically to 2+i.

(ii) The line y=x/2

7. Determine the poles , order of Pole of function and the residues at each pole:

(i) $f(z)=\dfrac{z^2}{(z-1)^2(z+2)}$ (ii) $f(z)=\dfrac{z^2}{(z-1)(z+2)^2}$

(iii) $f(z)=\dfrac{1-e^{2z}}{z^4}$ (iv) $f(z)=\dfrac{z^2}{(z-1)(z-2)(z-3)}$

8. Use calculus of residues to show that

(i) $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\sin\theta+a^2} d\theta = \frac{2\pi a^2}{1-a^2}, \quad a^2<1$

(ii) $\displaystyle\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2+b^2}}, \quad a>b>0$

(iii) $\displaystyle\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} d\theta = \frac{2\pi}{\sqrt{3}}$

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