Electrostatics in Vacuum

(CO-5 PART-I)

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Contents

The gradient of a scalar field

Definition: In a scalar field, the *maximum rate of change* of scalar function in space is known as gradient of the scalar field.

Gradient of a scalar is a *vector quantity*. Its direction is in which rate of change is *maximum*.

Let U(x, y, z) be a scalar function of position at the point of coordinates (x, y, z) . The gradient of scalar function U is defined as

$$\mathrm{grad}\,U \equiv \vec{\boldsymbol{\nabla}} U$$

$$\mathrm{grad}\,U = \frac{\partial U}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial U}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial U}{\partial z}\hat{\boldsymbol{k}}$$

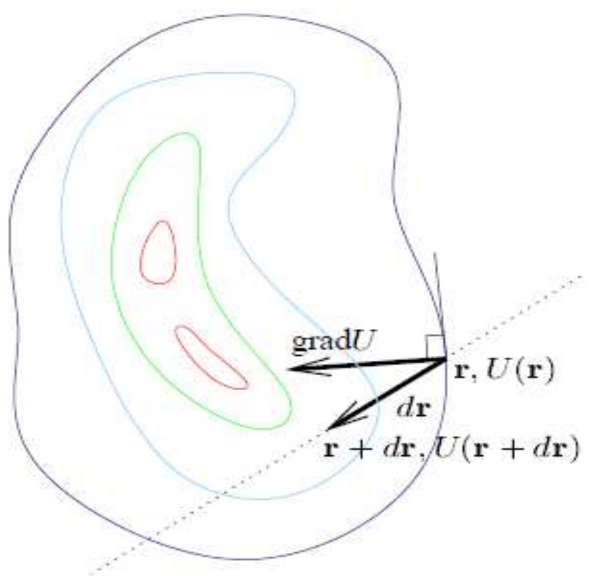
It is usual to define the **vector operator**

$$\vec{\boldsymbol{\nabla}} = \hat{\boldsymbol{\imath}}\,\frac{\partial}{\partial x} + \hat{\boldsymbol{\jmath}}\,\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\,\frac{\partial}{\partial z}$$

which is called “del” or “nabla”. Then

Significance of Gradient

The gradient of scalar field U is a *vector quantity* whose magnitude at any point is equal to the maximum rate of increase of U at that point and whose direction is along the normal to the surface at that point.



Example: The intensity of electric field $\vec{E}$ is a gradient of potential V (a scalar quantity) with negative sign.

$$\vec{E} = -\text{grad } V$$

Divergence of a vector field

- ***Definition***: The divergence of a vector field at any point is defined as the *amount of flux per unit volume diverging* from the point.

The divergence of a vector is a *scalar quantity*. the scalar or a dot product of $\vec{\nabla}$ with vector $\vec{a}$ is called divergence.

Let $\vec{a}$ be a vector function at each point (x, y, z) then

$$\operatorname{div} \vec{a} = \vec{\nabla} . \vec{a} = \left(\hat{\imath} \frac{\partial}{\partial x} + \hat{\jmath} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \vec{\mathbf{a}}$$

$$= \left(\hat{\imath} \frac{\partial}{\partial x} + \hat{\jmath} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) . \quad a_1 \hat{\imath} + a_2 \hat{\jmath} + a_3 \hat{k}$$

$$\boxed{\operatorname{div} \vec{\mathbf{a}} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}}$$

where $\vec{a} = a_1 \hat{\imath} + a_2 \hat{\jmath} + a_3 \hat{k},$

Significance of Divergence

At any point in the field divergence gives the *amount of flux per unit volume diverging* from that point.

- If divergence of any field is positive that means *a source* is placed inside the volume element.

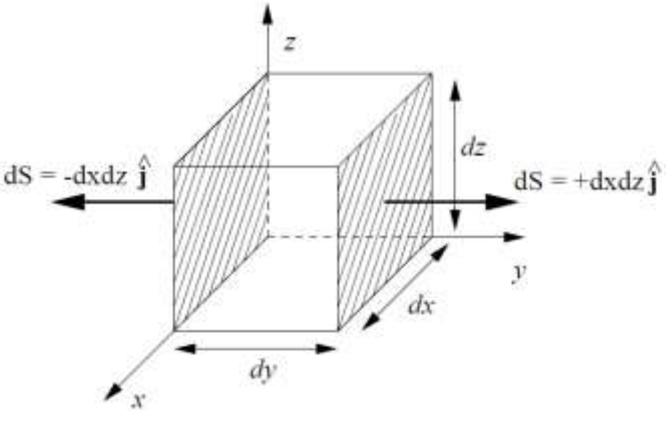
$$\text{div}\ \vec{a} = +\ (\text{positive})$$

- If divergence of any field is negative that means *a sink* is placed inside the volume element.

$$\text{div}\ \vec{a} = -\ (\text{negative})$$

- If divergence of any field is zero that neither source nor sink is inside the volume element. This type of vector is called *solenoidal vector* or field.

$$\boxed{\text{div}\ \vec{a} = 0\ (\text{Zero})}$$



Curl of a vector field

Definition:-The curl of a vector field is defined as the *maximum line integral* of the vector per unit area. It is a *vector quantity*, the direction is normal to the area surface.

The vector or a cross product of $\vec{\nabla}$ with vector $\vec{a}$ is called curl of vector a.

Let $\vec{a}$ be a vector function at each point (x, y, z) then

$$\text{Curl } \vec{a} = \vec{\nabla} \times \vec{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$$

$$= \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)\hat{k}$$

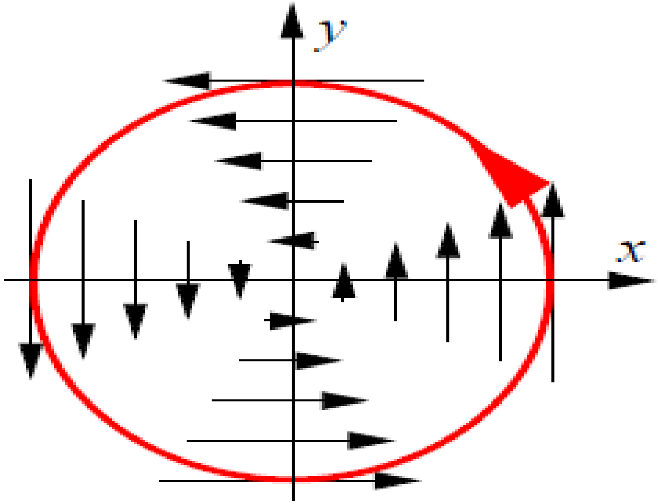
Where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$,

Significance of Curl

Curl of a vector represent the rotational of irrotational nature of a vector physically.

In other words curl of a vector measures the tendency of a vector field to spin about a given point or axis. for example velocity vector of water.

- If curl of vector is non zero then vector field is rotational.
- If curl of vector $\vec{a}$ is zero then vector field is known as irrotational field.



Curl $\vec{a} = 0$ (Zero)

Charge Densities

Charge densities are of *three types*:-

Linear charge density (λ):-It is defined as the charge distributed per unit *length* of a linear conductor. its unit is C/m.

$$\lambda = q/l = \int q\, dl$$ *where dl is an length element.*

Surface charge density(σ):- It is defined as the charge distributed per unit *area* of a conductor. its unit is C/m^2.

$$\sigma = q/a = \iint q\, da$$ *where da is an area element.*

Volume charge density(ρ):- It is defined as the charge distributed per unit *volume* of a conductor. its unit is C/m^3.

$$\rho = q/v = \iiint q\, dv$$ *where dv is an volume element.*

Gauss Divergence Theorem

Statement- The flux of a vector field $\vec{F}$ over any closed surface S is equal to the volume integral of the divergence of the vector field over the volume enclosed by a surface S.

$$\iint \vec{F}.\vec{dS} = \iiint \text{div}\vec{F}\, dV$$

or

$$\iint \vec{F}.\vec{dS} = \iiint (\vec{\nabla}.\vec{F})\, dV$$

This is used to convert the volume integral of the divergence of a vector field into surface integral & vice versa.

Stock's Theorem

Statement- The flux of the curl of a vector field $\vec{F}$ over any closed surface S of any shape is equal to the line integral of the vector field F over the boundary of that surface,

$$\boxed{\begin{gathered}\int \vec{F}.\vec{dl} = \iint \text{curl}\ \vec{F}.\vec{ds} \\ \text{or} \\ \int \vec{F}.\vec{dl} = \iint (\vec{\nabla} \times \vec{F}).\vec{ds}\end{gathered}}$$

This is used to convert the surface integral of the curl of a vector field into line integral & vice versa.