Chapter - 2

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UNIT - 2 NUMERICAL ANALYSIS - II

Numerical Differentiation

We have Newton Forward formula.

$$f(x_0+ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0 + \cdots$$

when $p = \frac{x-x_0}{h}$ where x = estimate value
x_0 = initial value
h = interval

Differentiate w.r.t. 'p' both sides

$$hf'(x_0+ph) = \Delta y_0 + \frac{(2p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{\lfloor 3}\Delta^3 y_0 + \cdots$$
$$+ \frac{1}{24}(4p^3-18p^2+22p+6)\Delta^4 y_0 + \frac{5p^4-40p^3+105p^2-100p+24}{120}\Delta^5 y_0$$

Again
$$h^2 f''(x_0+ph) = \Delta^2 y_0 + \frac{(6p-6)}{6}\Delta^3 y_0 + \frac{(12p^2-36p+22)}{24}\Delta^4 y_0$$

$$h^3 f'''(x_0+ph) = \Delta^3 y_0 + \frac{1}{12}(12p-18)\Delta^4 y_0 + \cdots$$

Hence,

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{\lfloor 3}\Delta^3 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6p-6)}{\lfloor 3}\Delta^3 y_0 + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{1}{12}(12p-18)\Delta^4 y_0 + \cdots\right]$$

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We have Newton Backward formula

$$f(x_n + ph) = y_n + p\nabla y_n + \frac{p(p+1)}{L2}\nabla^2 y_n + \cdots\cdots$$

$$\frac{p(p+1)(p+2)}{L3}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{L4}\nabla^4 y_n + \cdots$$

where $p = \frac{x - x_n}{h}$

x = estimate value

x_n = last value of x

h = Interval

Differentiate w.r.t. 'p' both sides

$$hf'(x_n+ph) = \nabla y_n + \frac{(2p+1)}{L2}\nabla^2 y_n + \frac{(3p^2+6p+2)}{L3}\nabla^3 y_n + \frac{4p^3+18p^2+22p+6}{L4}\nabla^4 y_n + \cdots$$

$$h^2 f''(x_n+ph) = \nabla^2 y_n + \frac{(6p+6)}{L3}\nabla^3 y_n + \frac{(12p^2+36p+22)}{L4}\nabla^4 y_n + \cdots$$

$$h^3 f''(x_n+ph) = \nabla^3 y_n + \frac{24p+36}{L4}\nabla^4 y_n + \cdots$$

Hence

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{(2p+1)}{L2}\nabla^2 y_n + \frac{(3p^2+6p+2)}{L3}\nabla^3 y_n + \frac{4p^3+18p^2+22p+6}{L4}\nabla^4 y_n + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{(6p+6)}{L3}\nabla^3 y_n + \frac{(12p^2+36p+22)}{L4}\nabla^4 y_n + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{24p+36}{L4}\nabla^4 y_n + \cdots\right]$$

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Q Find $f'(1.1)$ and $f''(1.1)$. Dec 2017

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0.0	0.1230	0.5540	1.2960	2.4320	4.000.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	0.0					
		0.128				
1.2	0.1230		0.298			
		0.426		0.018		
1.4	0.5540		0.316		0.062	
		0.742		0.08		-0.106
1.6	1.2960		0.396		-0.044	
		1.136		0.036		
1.8	2.4320		0.432			
		1.568				
2.0	4.000					

$$p = \frac{1.1 - 1.0}{0.2} = 0.5 \quad , h = 0.2.$$

$$f'(1.1) = \frac{1}{0.2}\left[\Delta y_0 + \frac{(2p-1)}{L2}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{L3}\Delta^3 y_0 + \cdots \right]$$

$$= \frac{1}{0.2}\left[0.128 + \frac{(2\times 0.5 - 1)}{L2} 0.298 + \frac{(3\times(0.5) - 6\times 0.5 + 2)}{L3} \times 0.018 \right.$$

$$+ \frac{1}{24}\left(4(0.5)^3 - 18(0.5)^2 + 22(0.5) - 6\right) \times 0.062.$$

$$\left. + \frac{1}{120}\left(5(0.5)^4 - 40(0.5)^3 - 105(0.5)^2 - 100(0.5) + 24\right) \times -0.106 \right]$$

$$= \frac{1}{0.2}\left[0.128 + 0.0015 + 0.0025 + 0.0503\right] = 0.9115.$$

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$$f''(1.1) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{6p-6}{13}\Delta^3 y_0 + \left(\frac{12p^2-36p+22}{24}\right)\Delta^4 y_0\right]$$

$$= \frac{1}{(0.2)^2}\left[0.298 + \frac{6(0.5)-6\times 0.013}{13} + \frac{12(0.5)^2-36\times 0.5+22\times 0.062}{24}\right.$$

$$= \frac{1}{(0.2)^2}\left[0.298 + (0.00415) + (-0.07875)\right.$$

$$= \frac{1}{(0.2)^2} \times 0.2551$$

$$= 1.3775.$$

*** Q. Find the first and second derivative of $f(x)$ at $x = 1.5$.

June 2008, June 2011

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.0	13.625	24.00	38.375	59.00

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375					
		3.625				
2.0	7.0		3			
		6.625		0.75		
2.5	13.625		3.75		0	
		10.375		0.75		0
3.0	24.00		4.5		0	
		14.375		0.75		
3.5	38.375		5.25			
		20.125				
4.0	59.00					

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To find $f'(x)$, at $x = x_0 = 1.5$

Using derivatone of forwerd diff. formula

Here $p = \frac{x - x_0}{h} = \frac{1.5 - 1.5}{0.5} = 0$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0\right]$$

$$= \frac{1}{0.5}\left[3.625 + \frac{1}{2}(3) + \frac{1}{3}(0.75)\right]$$

$$= \frac{2.375}{0.5}$$

$$\left(\frac{dy}{dx}\right)_{x=1.5} = 4.75 \quad \text{Ans}$$

and $$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0\right]$$

$$= \frac{1}{(0.5)^2}\left[3.00 - 0.75\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1.5} = 9.00 \quad \text{Ans}$$

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Q Given that Dec 2011, June 2013

x:	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=1.1$ and $x=1.6$

Sol. The difference table is given below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	7.989					
		0.414				
1.1	8.403		-0.036			
		0.378		0.006		
1.2	8.781		-0.030		-0.002	
		0.348		0.004		0.001
1.3	9.129		-0.026		-0.001	
		0.322		0.003		0.003
1.4	9.451		-0.023		0.002	
		0.299		0.005		
1.5	9.750		-0.018			
		0.281				
1.6	10.031					

(B) Find $y(1.1)$ by forward formula, $x_0=1.1$

$x=1.1$ Here $y_0=8.403$, $h=0.1$, $p=\frac{x-x_0}{h}=\frac{1.1-1.1}{0.1}=0$

$$\left(\frac{dy}{dx}\right)_{x=x_0}=\frac{1}{h}\left[\Delta y_0-\frac{1}{2}\Delta^2 y_0+\frac{1}{3}\Delta^4 y_0+\frac{1}{5}\Delta^5 y_0-\frac{1}{6}\Delta^6 y_0\right]$$

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$$\left(\frac{dy}{dx}\right)_{x=1.1} = \frac{1}{0.1}\left[0.378 + \frac{1}{2}(0.03) + \frac{1}{3}(0.004) + \frac{1}{4}(0.001) + \frac{1}{5}(0.003)\right]$$

$$= 3.9518$$

and $$\left(\frac{d^2y}{dx^2}\right)_{x=1.1} = \frac{1}{h^2}\left[\Delta^3 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.030 - 0.004 + \frac{11}{12}(-0.001) - \frac{5}{6}(0.003)\right]$$

$$= -3.742$$

Now we obtained $f'(x)$ and $f''(x)$ at $x = 1.6$ we use backward formula

$$\left(\frac{dy}{dx}\right)_{x=1.6} = \frac{1}{0.1}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \frac{1}{6}\nabla^6 y_n + \cdots\right]$$

$$= \frac{1}{0.1}\left[0.281 + \frac{1}{2}(-0.018) + \frac{1}{3}(0.005) + \frac{1}{4}(0.002 + \frac{1}{5}(0.003) + \frac{1}{6}(0.002)\right]$$

$$= 2.7476$$

and $$\left(\frac{d^2y}{dx^2}\right)_{x=1.6} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \frac{137}{180}\nabla^6 y_n + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.018 + 0.005 + \frac{11}{12}(0.002) + \frac{5}{6}(0.003) + \frac{137}{180}(0.002)\right]$$

$$\boxed{f''(1.6) = -0.7144}$$

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Numerical Integration

1. Trapezoidal Rule.

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots y_{n-1})\right]$$

(first term + last term) + 2 (sum of remaining term)

2. Simpson's One-Third Rule (1/3 Rule)

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+y_6+\cdots) + 4(y_1+y_3+y_5+\cdots)\right]$$

(first term + last term) + 2 (sum of even y_n) + 4 (sum of odd y_n)

3. Simpson's Three-Eight Rule (3/8 Rule)

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 2(y_3+y_6+y_9+\cdots) + 3(y_1+y_2+y_4+\cdots)\right]$$

(first + last term) + 2 (sum of 3 multiples) + 3 (sum of remaining)

Q. Evaluate $\int_{0.5}^{0.7} x^{1/2} e^{-x}\,dx$ approximately by Simpson 1/3 rule.

June-2003, Dec-2006

$x_0 = 0.5$

$x_0 + nh = 0.7 = 0.5 + 0.2$

$$h = \frac{0.7-0.5}{4} = \frac{0.2}{4} = 0.05$$

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x_0	$y = f(x) = x^{1/2} e^{-x}$
0.5	0.4288819
0.55	0.4278774
0.6	0.4251076
0.65	0.4208867
0.7	0.4154730

$$\int_{0.5}^{0.7} x^{1/2} e^{-x}\, dx = \frac{0.05}{3}\left[(0.4288819 + 0.4154730) + 2(0.4251076) + 4(0.4278774 + 0.4208867)\right].$$

$$= \frac{0.05}{3}\left[0.844292 + 0.8502152 + 4(0.8487641)\right] \quad = 3.3950565$$

$$= \frac{0.05}{3}(5.0895637)$$

$$= 0.084826.$$

Q. Using Simpson's 1/3 Rule, obtain $\int_{-3}^{3} x^4\, dx$ by taking seven ordinate. Compare it with exact value. NOV-2018

$$h = \frac{3+3}{6} = 1.$$

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x_0	$y = f(x) = x^4 dx$
-3	81
-2	16
-1	1
0	0
1	1
2	16
3.	81

$$\int_{-3}^{3} x^4 dx = \frac{1}{3}\left[(81+81) + 2(1+1) + 4(16+0+16)\right]$$

$$= \frac{1}{3}\left[162 + 4 + 128\right)$$

$$= \frac{194}{3} = \cancel{54.66}\ 98.6667.$$

Q. Use Simpson's 1/3rd Rule to find $\int_0^{0.6} e^{-x^2} dx$ by taken seven ordinate.

June-2009, June-2014

$f(x) = e^{-x^2}$

$x_0 = 0.$

$x_0 + nh = 0.6$

$nh = 0.6 - x_0$

$h = \frac{0.6}{6} = 0.1$

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x	$f(x) = e^{-x^2}$
0	1 (y_0)
0.1	0.99 (y_1)
0.2	0.96 (y_2)
0.3	0.9139 (y_3)
0.4	0.85214 (y_4)
0.5	0.7788 (y_5)
0.6	0.69767 (y_6)

$$\int_0^{0.6} e^{-x^2}\,dx = \frac{0.1}{3}\left(1+0.69767\right)+2\left(0.96+0.85214\right)+4\left(0.99+0.9139+0.7788\right)$$

$$= \frac{0.1}{3}\left(1.69767 + 3.62428 + 10.7308\right)$$

$$= \frac{0.1}{3} \times 16.05275$$

$$= 5.35091 = 0.53509$$

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Q. A river is 80 feet wide. The depth 'y' of the river at a distance 'x' from one bank is given by following table

x	0	10	20	30	40	50	60	70	80.
y	0	4	7	9	12	15	14	8	3.

Find approximately area of cross section of river using
(i) Simpson's 1/3 rule. Dec 2012
(ii) Trapezoidal rule

Sol Area of cross section

$$\int_0^{80} y\,dx$$

$$x_0 + nh = 80$$

$$nh = 80 - x_0 \qquad h = \frac{80}{8} = 10.$$

$$\int_0^{80} y\,dx = \frac{10}{3}\left[(y_0+y_8) + 2(y_2+y_4+y_6) + 4(y_1+y_3+y_5+y_7)\right]$$

$$= \frac{10}{3}\left[(0+3) + 2(7+12+14) + 4(4+9+15+8)\right]$$

$$= \frac{10}{3} \times (3 + 66 + 144)$$

$$= \frac{213 \times 10}{3}$$

$$= 710.$$

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80 By Trapezoidal Rule

$$\int_0^{80} y\,dx = \frac{10}{2}\left[(0+3) + 2(4+7+9+12+15+14+8)\right]$$

$$= \frac{10}{2} \times (3 + 138)$$

$$= \frac{1410}{2} = 705$$

Q. Evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\cdot d\theta$. Dec 2005, June 2011

by dividing the interval into 6 equal parts.

$x_0 = 0$, $x_0 + nh = \pi/2$

$0 + (6)h = \pi/2$

$h = \pi/12$.

$f(\theta) = \sqrt{\cos\theta}$

θ	$f(\theta) = \sqrt{\cos\theta}$
0	1
$\pi/12$	0.982815
$\pi/6$	0.9306
$\pi/4$	0.840896
$\pi/3$	0.7071
$5\pi/12$	0.508742
$\pi/2$	0

$$\int_0^{\pi/2} \sqrt{\cos\theta}\cdot d\theta = \frac{\pi}{36}\left[(1+0) + 2(0.9306 + 0.7071) + 4(0.982815 + 0.840896 + 0.508742)\right]$$

$$= \frac{\pi}{36}\left[1 + 3.2754 + 9.329812\right]$$

$$= \frac{\pi}{36} \times 13.605212$$

$$= 1.187278724.$$

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Q. Find an approximate value of $\log_e 5$ by calculating to four decimal place by Simpson 1/3 rule.
$\int_0^5 \frac{dx}{4x+5}$ dividing the range into 10 equal parts
June. 2004, Dec 2008, B

Here $f(x) = \frac{1}{4x+5}$

$x_0 = 0$, $x_0 + nh = 5$

$0 + 10h = 5$

$h = \frac{5}{10} = 0.5$

x	$f(x)$
0	1/5
0.5	1/7
1	1/9
1.5	1/11
2	1/13
2.5	1/15
3	1/17
3.5	1/19
4	1/21
4.5	1/23
5	1/25

$$\int_0^5 \frac{dx}{4x+5} = \frac{0.5}{3}\left[(1/5 + 1/25) + 2\left(\frac{1}{9}+\frac{1}{13}+\frac{1}{17}+\frac{1}{21}\right) + 4\left(\frac{1}{7}+\frac{1}{11}+\frac{1}{15}+\frac{1}{19}+\frac{1}{23}\right)\right]$$

$$= \frac{0.5}{3}\left[0.24 + 0.588953 + 1.583610\right)$$

$$= \frac{0.5}{3} \times 2.415123$$

$$= 0.40253.$$

$$\int_0^5 \frac{dx}{4x+5} = 0.40253$$

$$\frac{1}{4}\left[\log(4x+5)\right]_0^5 = 0.40253$$

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$$\frac{1}{4}\left[\log 25 - \log 5\right] = 0.40253$$

$$\frac{1}{4}\left[\log\left(\frac{25}{5}\right)\right] = 0.40253$$

$$\frac{1}{4}\left[\log 5\right] = 0.040253$$

$$\log 5 = 4 \times 0.40257$$

$$\log 5 = 1.61010.$$

* SOLUTION OF SIMULTANEOUS LINEAR ALGEBRIC EQUATION

Direct Method.

1. Gauss Elimination Method.
2. Gauss Jordan Method.
3. Crout's ~~Doig~~ Triangularization Method
   or
   Cholesky Method.

★ Gauss Elimination Method.

Q. Solve the following system eqn by Gauss elimination method. Dec-2001, June 2017

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

$x = 1$
$y = 2$
$z = 3.$

Ans

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Given eqn can be written as

$$Ax = B.$$

where,

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

Augumented Matrix.

$$[A : B] = \left[\begin{array}{ccc:c} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$R_1 \leftarrow R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 9 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & -3 \\ 0 & -2 & 0 & -4 \end{array}\right]$$

$R_2 \rightarrow R_2/3$

$R_3 \rightarrow R_3/2$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1/3 & -1 \\ 0 & -1 & 0 & -2 \end{array}\right]$$

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$R_3 \longrightarrow R_3 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1/3 & -1 \\ 0 & 0 & -1/3 & -1 \end{array}\right]$$

$$x + y + z = 6 \quad \text{— (1)}$$
$$-y + \frac{1}{3}z = -1 \quad \text{— (2)}$$
$$-\frac{1}{3}z = -1 \quad \text{— (3).}$$

from (3).

$z = 3$.

put $z = 3$ in (2)

$y = 2$.

& put $y = 2$, $z = 3$ in (1)

$x = 1$.

Q. Solve by Gauss elimination method, June-2002

$10x + y + 2z = 13$
$3x + 10y + z = 14$
$2x + 3y + 10z = 15$

$x = 1$
$y = 1$
$z = 1$

Given eqn can be written as

$Ax = B$

where, $A = \begin{bmatrix} 10 & 1 & 2 \\ 3 & 10 & 1 \\ 2 & 3 & 10 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 13 \\ 14 \\ 15 \end{bmatrix}$

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Augmented matrix

$$[A:B] = \left[\begin{array}{ccc|c} 10 & 1 & 2 & 13 \\ 3 & 10 & 1 & 14 \\ 2 & 3 & 10 & 15 \end{array}\right]$$

$R_1 \leftarrow R_2$

$R_1 \rightarrow R_1 - 3R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -29 & -1 & -29 \\ 3 & 10 & 1 & 14 \\ 2 & 3 & 10 & 15 \end{array}\right]$$

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -29 & -1 & -29 \\ 0 & 97 & 4 & 101 \\ 0 & 61 & 12 & 73 \end{array}\right]$$

$R_2 \rightarrow R_2/97$

$$\sim \left[\begin{array}{ccc|c} 1 & -29 & -1 & -29 \\ 0 & 1 & 4/97 & 101/97 \\ 0 & 61 & 12 & 73 \end{array}\right]$$

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$R_3 \to R_3 - 61R_2$

$$\sim \begin{bmatrix} 1 & -29 & -1 & -29 \\ 0 & 1 & 4/97 & 101/97 \\ 0 & 0 & 920/97 & 920/97 \end{bmatrix}$$

$$x - 29y - z = -29$$
$$y + 4/97 z = 101/97$$
$$920/97 z = 920/97$$

$z = 1$
$y = 1$
$x = 1$. } 1½

Q. Solve by Gauss elimination method

$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

$x = 7$
$y = -9$
$z = 5$

June. 2012

Augumented matrix

$$[A : B] = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right].$$

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1 4 9 16
3 2 3 19
2 1 1 10

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$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -7 & -17 & -22 \end{array}\right]$$

$19-3\times16$
$19-48$

$R_3 \to R_3 + 7$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -46 \\ 0 & 0 & -10 & -15 \end{array}\right]$$

$-7-7$
$-14+7$
$-22+7$

$x + 4y + 9z = 16$
$-10y - 24z = -46$
$-10z = -15$

$R_2 \to R_2/-10$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 1 & 2.4 & 3 \\ 0 & -7 & -17 & -22 \end{array}\right]$$

$-10/-10$
$-24/-10$
$-46/10$

$R_3 \to R_3 + 7R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 1 & 2.4 & 3 \\ 0 & 0 & -1/5 & -1 \end{array}\right]$$

$-7+7\cdot1$
$-17+7\times24$
$-17+7\times24$

$x+4y+9z=16$
$y+2.4z=4.6$
$-\frac{1}{5}z=1$
Solve fm's
$z=5, y=-9, x=7$

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Gauss Jordan Method

Q. Apply Gauss Jordan method to find the solution Dec 2014

$$10x + y + z = 12$$
$$2x - 10y + z = 13$$
$$x + y + 5z = 7$$

$$[A:B] = \left[\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 2 & -10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 2 & -10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 10R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & -12 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{array}\right]$$

$R_2 \rightarrow R_2/-12$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 1 & 9/12 & 1/12 \\ 0 & -9 & -49 & -58 \end{array}\right]$$

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~~$R_3 \to R_3 + 9R_2$~~

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 1 & 9/12 & 1/12 \\ 0 & -9 & -49 & -58 \end{bmatrix}$$

$R_3 \to R_3 + 9R_2$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 1 & 9/12 & 1/12 \\ 0 & 0 & -169/4 & -687/12 \end{bmatrix}$$

$R_3 \to R_3 / -169/4.$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 1 & 9/12 & 1/12 \\ 0 & 0 & 1 & 229/169 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 17/4 & 87/12 \\ 0 & 1 & 9/12 & 1/12 \\ 0 & 0 & 1 & 229/169 \end{bmatrix}$$

$R_2 \to R_2 - \frac{9}{12} R_3.$

$$\sim \begin{bmatrix} 1 & 0 & 17/4 & 87/12 \\ 0 & 1 & 0 & -245/48 \\ 0 & 0 & 1 & 229/169 \end{bmatrix} -473/507$$

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$R_3 \rightarrow R_1 - \frac{51}{12} R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 587/507 \\ 0 & 1 & 0 & -473/507 \\ 0 & 0 & 1 & 229/169 \end{array}\right]$$

$x = \frac{587}{507} = 1.1577.$

$y = \frac{-473}{507} = -0.93.$

$z = \frac{229}{169} = 1.355$

Q. Solve by Gauss Jordan method.

$$10x + y + z = 9$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

$x = 25/36$

$y = 37/36$

$z = 37/36$

June 2022, Dec 2017

$$[A:B] = \left[\begin{array}{ccc|c} 10 & 1 & 1 & 9 \\ 1 & 10 & 1 & 12 \\ 1 & 1 & 10 & 12 \end{array}\right]$$

$R_3 \leftrightarrow R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 10 & 12 \\ 1 & 10 & 1 & 12 \\ 10 & 1 & 1 & 9 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 10 & 12 \\ 10 & 1 & 1 & 9 \\ 1 & 10 & 1 & 12 \end{array}\right]$$

12.

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$R_2 \to R_2 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 10 & 12 \\ 0 & 9 & -9 & 0 \\ 10 & 1 & 1 & 9 \end{bmatrix}$$

$R_3 \to R_3 - 10R_1$

$$\sim \begin{bmatrix} 1 & 1 & 10 & 12 \\ 0 & 9 & -9 & 0 \\ 0 & -9 & -99 & -111 \end{bmatrix}$$

1 - 10 · 1
1 - 10
1 - 10 · 10
9 - 10 · 12
9 - 120

~~$R_2 \to R_2$~~ $R_2 \to R_2/9$.

$$\sim \begin{bmatrix} 1 & 1 & 10 & 12 \\ 0 & 1 & -1 & 0 \\ 0 & -9 & -99 & -111 \end{bmatrix}$$

$R_3 \to R_3 + 9R_2$

$$\sim \begin{bmatrix} 1 & 1 & 10 & 12 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -108 & -111 \end{bmatrix}$$

-9 + 9 · 1
-99 + 9 - 9
-99

$R_3 \to R_3/-108$

$$\sim \begin{bmatrix} 1 & 1 & 10 & 12 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 37/36 \end{bmatrix}$$

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$R_1 \to R_1 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 11 & 12 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 37/36 \end{bmatrix}$$

$R_2 \to R_2 + R_3$

$$\sim \begin{bmatrix} 1 & 0 & 11 & 12 \\ 0 & 1 & 0 & 37/36 \\ 0 & 0 & 1 & 37/36 \end{bmatrix}$$

$R_1 \to R_1 - 11R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 25/36 \\ 0 & 1 & 0 & 37/36 \\ 0 & 0 & 1 & 37/36 \end{bmatrix}$$

$x = 25/36$

$y = 37/36$

$z = 37/36$

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✱ Crout's Triangularization Method
or
Choleskey Method
or
Factorization Method.

We have $AX = B$ — ①

where, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & b_{22} & b_{23} \\ a_{31} & b_{32} & b_{33} \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

We know that
$A = LU$ — ②.

where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$

and $U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$

Then ① becomes
$L(UX) = B$
Suppose $UX = V$
where $V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

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Then $LV = B$

We find V and put V in $UX = V$

We get value of x, y, z.

Q- Apply Factorization method to solve eqn (Dec 2004, Dec 2007, Dec 2012)

$$3x + 2y + 2z = 4$$
$$2x + 3y + z = 5$$
$$3x + 4y + z = 7$$

We have $AX = B$

where, $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$

Now, we know that

$A = LU$

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

- $u_{11} = 3$
- $u_{12} = 2$
- $u_{13} = 7$

$l_{21} \cdot u_{11} = 2$

$l_{21} \cdot 3 = 2$

$l_{21} = 2/3$

$l_{21} \cdot u_{12} + u_{22} = 3$

$\frac{2}{3} \cdot 2 + u_{22} = 3$

$u_{22} = 5/3$

$\frac{4}{3} + u_{21} = 3 - \frac{4}{3}$

$y = \frac{4}{3} \cdot \frac{3}{5}$

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$l_{21} \cdot u_{13} + u_{23} + 0 = 1$

$\frac{2}{3} \cdot 7 + u_{23} = 1$

$\frac{14}{3} - 1$

$1 - \frac{14}{3}$

$\frac{3-14}{3}$

$$\boxed{u_{23} = \frac{-11}{3}}$$

$l_{31} \cdot u_{11} = 3$

$$\boxed{l_{31} = 1}$$

$l_{31} \cdot u_{12} + l_{32} \cdot u_{22} = 4$

$1 \cdot 2 + l_{32} \cdot \frac{5}{3}$

$l_{32} \cdot \frac{5}{3} = 4 - 2$

$l_{32} = 2 \cdot \frac{3}{5} = \frac{6}{5}$

$$\boxed{l_{32} = \frac{6}{5}}$$

$l_{31} \cdot u_{13} + l_{32} \cdot u_{23} + u_{33} = 1$

$1 \cdot 7 + \frac{6}{5} \cdot \frac{-11}{3} + u_{33} = 1$

$7 + \frac{-66}{15} + u_{33} = 1$

$\frac{13}{5}$

$u_{33} = -\frac{8}{5}$

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$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

~~$v_1 = 1$~~

$LV = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$\boxed{v_1 = 4}$

$\frac{2}{3} v_1 + v_2 = 5$

$\frac{2}{3} \cdot 4 + v_2 = 5$ ⇒ $v_2 = 5 - \frac{8}{3} = \frac{15-8}{3} = \frac{7}{3}$

$\boxed{v_2 = \frac{7}{3}}$

$v_1 + \frac{6}{5} v_2 + v_3 = 7$

$4 + \frac{6}{5} \times \frac{7}{3} + v_3 = 7$

$v_3 = 7 - 4 - \frac{42}{15} = \frac{1}{5}$

$\boxed{v_3 = \frac{1}{5}}$

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Hence, $UX = V$

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7/5 \\ 1/5 \end{bmatrix}$$

$$3x + 2y - 7z = 4$$

$$\frac{5}{3}y - \frac{11}{3}z = 7/3.$$

$$\frac{-8}{5}z = \frac{1}{5}.$$

$$\boxed{z = \frac{-1}{8}}$$

$$\boxed{y = \frac{9}{8}}$$

$$\boxed{x = \frac{7}{8}}$$

Q. Solve by Crout's Method. Dec 2013, June 2016, De 2016

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14.$$

Sol Given System in Matrix form $AX = B$

Let

$$A = LU$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

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∴ Since $LU = A$ we get

$u_{11} = 10$

$l_{21} \cdot u_{11} = 2$

$l_{21} = 1/5$

$l_{31} \cdot u_{11} = 2$

$l_{31} = 1/5$

$u_{12} = 1$

$l_{21} \cdot u_{12} + u_{22} = 10$

$u_{22} = 49/5$

$l_{31} \cdot u_{12} + l_{32} \cdot u_{22} = 2$

$l_{32} = \frac{9}{49}$

$u_{13} = 1$

$l_{21} \cdot u_{13} + u_{23} = 1$

$u_{23} = 4/5$

$l_{31} \cdot u_{13} + l_{32} \cdot u_{23} + u_{33} = 10$

$u_{33} = \frac{473}{49}$

Suppose $UX = V$

where $V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

then $AX = B \Rightarrow (LU)X = B$

$L(UX) = B$

$LV = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1 & 0 \\ 1/5 & 9/49 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix}$$

$v_1 = 12$

$v_2 = 53/5$

$v_3 = \frac{473}{49}$

$\therefore \frac{1}{5} v_1 + v_2 = 13$

$\Rightarrow \frac{1}{5} v_1 + \frac{9}{49} v_2 + v_3 = 14$

Hence $UX = V$ becomes.

$$\begin{bmatrix} 10 & 1 & 1 \\ 0 & 49/5 & 4/5 \\ 0 & 0 & 473/49 \end{bmatrix} \begin{bmatrix} x \\ y \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 53/5 \\ \frac{473}{49} \end{bmatrix}$$

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$10x + y + z = 12$ — (i)

$\frac{49}{5}y + \frac{4}{5}z = \frac{53}{5}$ — (ii)

$\left(\frac{473}{49}\right)z = \frac{473}{49}$ — (iii)

from (iii)

$\boxed{z = 1}$

Put $z = 1$ in (ii)

$\frac{49}{5}y + \frac{4}{5}(1) = \frac{53}{5}$

$\frac{49}{5}y = \frac{53}{5} - \frac{4}{5} = \frac{49}{5}$

$\boxed{y = 1}$

Put $y = 1$ and $z = 1$ in (i)

$10x + 1 + 1 = 12$

$10x = 12 - 2$

$10x = 10$

$\boxed{x = 1}$

Hence $\boxed{x = 1,\ y = 1,\ z = 1}$ Ans.

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Iterative Method.

1. Gauss- Seidal Method.
2. Relaxion Method.
3. Gauss- Jacobi Method.

★ Gauss- Seidal Method :-

Q. Solve by Gauss- Seidal Method

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

$x = 2.4254$
$y = 3.5730$
$z = 1.92595$

Dec-2002, Dec2006, June-2008, June-2011, Dec2011, June 2013, June-2017, NOV-2018

Given, $x = \frac{1}{27}[85 - 6y + z]$

$y = \frac{1}{15}[72 - 6x - 2z]$

$z = \frac{1}{54}[110 - x - y]$.

Here $|a_{11}| > |a_{12}| + |a_{13}|$

$|a_{22}| > |a_{21}| + |a_{23}|$

$|a_{33}| > |a_{31}| + |a_{32}|$

First approximation

We start $y = z = 0$ and using most recent value of x, y & z.

$$x^{(1)} = \frac{1}{27}[85 - 6(0) + 0] = \frac{85}{27} = 3.148.$$

$$y^{(1)} = \frac{1}{15}[72 - 6(3.148) - 2(0)] = 3.5408.$$

$$z^{(1)} = \frac{1}{54}[110 - 3.148 - 3.5408] = 1.91317$$

2nd approximation

$$x^{(2)} = \frac{1}{27}[85 - 6(3.5408) + 1.913) = 2.4321$$

$$y^{(2)} = \frac{1}{15}[72 - 6(2.4321) - 2(1.91317)] = 3.572$$

$$z^{(2)} = \frac{1}{54}[110 - 2.4321 - 3.572] = 1.92585$$

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3rd approximation

$$x^{(3)} = \frac{1}{27}\left[85 - 6\times 3.572 + 1.925\right] = 2.4256$$

$$y^{(3)} = \frac{1}{15}\left[72 - 6\times 2.4321 - 2\times 1.925\right] = 3.573$$

$$z^{(3)} = \frac{1}{54}\left[110 - 2.4256 - 3.573\right] = 1.9259.$$

Here, 2nd and 3rd approximation are same,

so,

$$\left.\begin{aligned} x &= 2.4256, \\ y &= 3.573 \\ z &= 1.9259 \end{aligned}\right\} \text{Ans}$$

Q Solve by Gauss - Seidal Method Dec 2003, June 2006

$$83x + 11y - 4z = 95$$
$$7x + 52y + 13z = 104$$
$$3x + 8y + 29z = 71$$

Here

$|a_{11}| > |a_{12}| + |a_{13}|$, $x = \frac{1}{83}\left[95 - 11y + 4z\right]$ $\quad y = \frac{1}{52}\left[104 - 7x + 13z\right]$

$|a_{22}| > |a_{21}| + |a_{23}|$, $y = \frac{1}{7}\left[104 - 52y - 13z\right]$ $\quad z = \frac{1}{29}\left[71 - 3x - 8y\right]$

$|a_{33}| > |a_{31}| + |a_{32}|$, $z = \frac{1}{3}\left[71 - 8x - 29\right]$

1st.

$$x^{(1)} = \frac{1}{83}\left[95 - 11(0) + 4(0)\right]$$

$$= 1.1445$$

$$y^{(1)} = \frac{1}{52}\left[104 - 7(1.445) - 13(0)\right]$$

$$= 1.8459$$

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$$z^{(1)} = \frac{1}{29}\left[71 - 3\times 1.1445 - 8\times 1.8459\right)$$

$$= 1.8206.$$

2nd. $$x^{(2)} = \frac{1}{83}\left[95 - 11(1.8459) + 4(1.8206)\right]$$

$$= \cancel{0.8122}\ 0.9876$$

$$y^{(2)} = \frac{1}{52}\left[104 - 7\times 0.9876 - 13\times 1.8206\right]$$

$$= \cancel{1.4355}\ 1.88524$$

$$z^{(2)} = \frac{1}{29}\left(71 - 3\times 0.9876 - 8\times 1.88524\right)$$

$$= 1.96397$$

3rd. $$x^{(3)} = \frac{1}{83}\left[95 - 11(1.88524) + 4(1.96397)\right)$$

$$= 1.05564$$

$$y^{(3)} = \frac{1}{52}\left(104 - 7\times 1.05564 - 13\times 1.96397\right)$$

$$= 1.3669$$

$$z^{(3)} = \frac{1}{29}\left[71 - 3\times 1.05564 - 8\times 1.3669\right]$$

$$= 1.96199.$$

Here 2nd & 3rd approximation are same,
so, $x = 1.05564$, $y = 1.3669$, $z = 1.96199$ (35)

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Q. Solve the by Gauss-Seidal Method.

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 + 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

$x_1 = 1.0777$

$x_2 = -0.4768$

$x_3 = -0.4368$

$x_4 = 10.$

June 2004, Dec 2006, June 2009

$x_1 = \frac{1}{10}[3 + 2x_2 + x_3 + x_4]$

$x_2 = \frac{1}{10}[15 + 2x_1 + x_3 + x_4]$

$x_3 = \frac{1}{10}[27 + x_1 + x_2 + 2x_4]$

$x_4 = \frac{1}{10}[-9 + x_1 + x_2 + 2x_3]$.

Here $|a_{11}| > |a_{12}| + |a_{13}| + |a_{14}|$

$|a_{22}| > |a_{21}| + |a_{23}| + |a_{24}|$

$|a_{33}| > |a_{31}| + |a_{32}| + |a_{34}|$

$|a_{44}| > |a_{41}| + |a_{42}| + |a_{43}|$

1st.

$x_1^{(1)} = \frac{1}{10}[3 + 3(0) + 0 + 0]$

$= 0.3$

$x_2^{(1)} = \frac{1}{10}[15 + 2(0.3) + 0 + 0]$

$= 1.56$

$x_3^{(1)} = \frac{1}{10}[27 + 0.3 + 1.56 + 2(0)]$

$= 2.886$

$x_4^{(1)} = \frac{1}{10}[-9 + 0.3 + 1.56 + 2(2.886)]$

$= -0.1368$

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2nd. $x_1^{(2)} = \frac{1}{10}[3 + 2(1.56) + 2.886 - 0.1368]$

$= 0.8906.$

$x_2^{(2)} = \frac{1}{10}[15 + 0(1.04292) + 2.886 - 0.1368]$

$= 1.956.$

$x_3^{(2)} = \frac{1}{10}[27 + 1.04292 + 1.77492 + 2(-0.13)]$

$= 2.96$

$x_4^{(2)} = \frac{1}{10}[-9 + 0.8906 + 1.956 + 2(2.958)]$

$= -0.02.$

3rd. $x_1^{(3)} = 0.9852$

$x_2^{(3)} = 1.9908$

$x_3^{(3)} = 2.99358$

$x_4^{(3)} = -0.00382$

4th $x_1^{(4)} = 0.99$

$x_2^{(4)} = 1.99$

$x_3^{(4)} = 2.99$

$x_4^{(4)} = -0.004.$

Here, 3rd & 4th approximation are same,

so, $x_1 = 0.99$, $x_2 = 1.99$, $x_3 = 2.99$

$x_4 = -0.004.$

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(2) Relaxation Method

Q Solve the following equation by relaxation Method.

$$10x - 2y - 2z = 6$$
$$-x + 10y - 2z = 7$$
$$-x - y + 10z = 8$$

The Residuals R_x, R_y and R_z are given as

$$R_x = 6 - 10x + 2y + 2z$$
$$R_y = 7 + x - 10y + 2z$$
$$R_z = 8 + x + y - 10z$$

The table of operations is

	δR_x	δR_y	δR_z
$\delta x = 1$	-10	-1	1
$\delta y = 1$	2	-10	1
$\delta z = 1$	(2)	(2)	(-10)

multiply, multiply, multiply, add, add

The Relaxation table is:

	R_x	R_y	R_z
	Add	Add	
$x = y = z = 0$	6 ↓ equal to	7 ↓ equal to	(8) ↓ equal to
$\delta z = \frac{8}{10} = 0.8$	7.6	(8.6)	0
$\delta y = \frac{8.6}{10} = 0.86$	(9.32)	0	0.86
$\delta x = \frac{9.32}{10} = 0.932$	0	0.932	(17.92)
		(38)	

$\delta z = \frac{1.2792}{10} = 0.1792$	0.3584	1.2904	0
$\delta y = \frac{1.2904}{10} = 0.12904$	0.61648	0	0.12904
$\delta x = \frac{0.61648}{10} = 0.061648$	0	0.061648	0.190688
$\delta z = \frac{0.190688}{10} = 0.0190688$	0.0381376	0.0997856	0
$\delta y = \frac{0.0997856}{10} = 0.00190688$	~~0.0194~~ 0.05809472	0	0.00190688
$\delta x = \frac{0.05809472}{10} = 0.005809472$	0	0.005809472	0.00771635
$\delta z = \frac{0.00771635}{10} = 0.000771635$	0.01543270	0.0212421	0
$\delta y = \frac{0.0212421}{10} = 0.00212421$	0.004248435	0	0.00212421
$\delta x = \frac{0.004248435}{10} = 0.0004248435$	0	0.0004248435	0.0002549035

$x = \Sigma x = 0.932 + 0.061648 + 0.005809472 + 0.0004248435$
$= 1.000566 \approx 1$

$y = \Sigma y = 0.86 + 0.12904 + 0.00190688 + 0.00212421$
$= 0.99307 \approx 1$

$z = \Sigma z = 0.8 + 0.1792 + 0.0190688 + 0.000771635$
$= 0.999040 \approx 1$

Hence $x = y = z = 1$ ✓

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Q. Solve by Relaxation method

$$3x + 9y - 2z = 11$$
$$4x + 2y + 13z = 24$$
$$4x - 4y + 3z = -8$$

$$|3| \not> |9| + |2|$$
$$|2| \not> |4| + |13|$$
$$|7| \not> |4| + |4|$$

This doesn't satisfy the condition.
So, it can't be solved by Relaxation method.

Q. Solve by Relaxation Method.

$$10x - 2y - 3z = 205$$
$$-2x + 10y - 2z = 154$$
$$-2x - y + 10z = 120.$$

$x = 32$
$y = 26$
$z = 21$

The Residuals R_x, R_y, & R_z are given as

$$R_x = 205 - 10x + 2y + 3z$$
$$R_y = 154 + 2x - 10y + 2z$$
$$R_z = 120 + 2x + y - 10z$$

The table of operation is

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	δR_x	δR_y	δR_z
$\delta x = 1$	-10	2	2
$\delta y = 1$	2	-10	1
$\delta z = 1$	3	2	-10

The Relaxation table is

	R_x	R_y	R_z
$x=y=z=0$	(205)	154	120.
$\delta x = \frac{205}{10} = 20.5$	0	(195)	161.
$\delta y = \frac{195}{10} = \frac{195}{19.5}$	~~3.9~~ 39	~~175.5~~ 0	~~162.95~~ (180.5)
$\delta z = \frac{180.5}{10} = 18.05$	(93.15)	36.1	0.
$\delta x = \frac{93.15}{10} = 9.315$	0	(54.73)	18.63
$\delta y = \frac{54.73}{10} = 5.473$	10.946	0	(24.103)
$\delta z = \frac{24.103}{10} = 2.4103$	(18.1769)	4.8206	0
$\delta x = \frac{18.1769}{10} = 1.81769$	0	(8.45598)	3.63538
$\delta y = 0.845598$	1.691196	0	(4.480978)
$\delta z = 0.4480978$	3.0354894	0.8961956	0

$x = \Sigma \delta x = 31.63269 \sim 32$
$y = \Sigma \delta y = 25.818598 \sim 26$
$z = \Sigma \delta z = 20.9083978 \sim 21$

Ans

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Gauss – Jacobi Method.

Q. Solve Gauss Jacobi Method.

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110.$$

Here, $|a_{11}| > |a_{12}| + |a_{13}|$
$|a_{22}| > |a_{21}| + |a_{23}|$
$|a_{33}| > |a_{31}| + |a_{32}|$.

$$x = \frac{1}{27}[85 - 6y + z]$$

$$y = \frac{1}{15}[72 - 6x - 2z]$$

$$z = \frac{1}{54}[110 - x - y]$$

Starting with initial approximation $x = y = z = 0$.

First Approx.

$$x^{(1)} = \frac{85}{27} = 3.148$$

$$y^{(1)} = \frac{72}{15} = 4.8$$

$$z^{(1)} = \frac{110}{54} = 2.037$$

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2nd. $x^{(2)} = \frac{1}{27}[85 - 6(4.8) + 2.037]$

$= 2.1569$

$y^{(2)} = \frac{1}{15}[72 - 6 \times 3.148 - 2 \times 2.037]$

$= 3.2692$

$z^{(2)} = \frac{1}{54}[110 - 3.148 - 4.8]$

$= 1.889$

3rd. $x^{(3)} = \frac{1}{27}[85 - 6 \times 3.2692 + 1.889]$

$= 2.4915$

$y^{(3)} = \frac{1}{15}[72 - 6 \times 2.1569 - 2 \times 1.889]$

$= 3.68537$

$z^{(3)} = \frac{1}{54}[110 - 2.1569 - 3.2692]$

$= 1.93655$

4th. $x^{(4)} = \frac{1}{27}[85 - 6 \times 3.68537 + 1.93655]$

$= 2.4009$

$y^{(4)} = \frac{1}{15}[72 - 6 \times 2.4915 - 2 \times 1.93655]$

$= 3.54519$

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$$z^{(4)} = \frac{1}{54}\left(110 - 2.4915 - 3.68577\right)$$

$$= 1.92265$$

5th. $x^{(5)} = \frac{1}{27}\left[85 - 6 \times 3.54519 + 1.92265\right]$

$$= 2.4315$$

$$y^{(5)} = \frac{1}{15}\left[72 - 6 \times 2.4009 - 2 \times 1.92265\right]$$

$$= 3.58328$$

$$z^{(5)} = \frac{1}{54}\left[110 - 2.4009 - 3.54519\right]$$

$$= 1.92692$$

6th $x^{(6)} = \frac{1}{27}\left[85 - 6 \times 2.4009 + \right.$

$$x^{(6)} = \frac{1}{27}\left[85 - 6 \times 3.58328 + 1.92692\right].$$

$$= 2.42323$$

$$y^{(6)} = \frac{1}{15}\left(72 - 6 \times 2.4315 - 2 \times 1.92692\right)$$

$$= 3.5704$$

$$z^{(6)} = \frac{1}{54}\left(110 - 2.4315 - 3.58328\right)$$

$$= 1.92688.$$

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7th. $x^{(7)} = \frac{1}{27}[85 - 6 \times 3.5704 + 1.9258]$

$= 2.426$

$y^{(7)} = \frac{1}{15}(72 - 6 \times 2.42323 - 2 \times 1.9258)$

$= 3.573$

$z^{(7)} = \frac{1}{54}(110 - 2.42323 - 3.5704)$

$= 3.573$ 1.926

Here, 6th & 7th approximation are same.

So, $x = 2.42$
$y = 3.57$
$z = 1.926$ } ≈

Q. Solve by Gauss - Jacobi Method.

$10x - 5y - 2z = 3.$
$4x - 10y + 3z = -3.$
$x + 6y + 10z = -3.$

$x = 0.341$
$y = 0.285$
$z = -0.505$

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$x = \frac{1}{10}[3 + 5y + 2z)$

$y = \frac{1}{10}[+3 + 4x + 3z].$

$z = \frac{1}{10}[-3 - x - 6y.].$

Starting with initial approximation $x = y = z = 0.$

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1st $x^{(1)} = \frac{1}{10}\left[3 + 5(0) + 2(0)\right]$

$= 0.3$

$y^{(1)} = \frac{1}{10}\left[3 + 4(0) + 3(0)\right]$

$= 0.3$

$z^{(1)} = \frac{1}{10}\left[-3 - 0 - 6(0)\right]$

$= -0.3$

2nd. $x^{(2)} = \frac{1}{10}\left[3 + 5(0.3) + 2(-0.3)\right]$

$= 0.39$

$y^{(2)} = \frac{1}{10}\left[3 + 4(0.3) + 3(-0.3)\right]$

$= 0.33$

$z^{(2)} = \frac{1}{10}\left(-3 - 0.3 - 6(0.3)\right]$

$= -0.51$

3rd $x^{(3)} = \frac{1}{10}\left[3 + 5(0.33) + 2(-0.51)\right]$

$= 0.363$

$y^{(3)} = \frac{1}{10}\left[3 + 4(0.39) + 3(-0.51)\right)$

$= 0.303$

$z^{(3)} = \frac{1}{10}\left(-3 - 0.39 - 6(0.33)\right]$

$= -0.537$

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4th. $x^{(4)} = \frac{1}{10}[3 + 5(0.303) + 2(-0.537)$

$= 0.3441$

$y^{(4)} = \frac{1}{10}[3 + 4(0.3441) + 3(-0.537)]$

$= 0.2841$

$z^{(4)} = \frac{1}{10}(-3 - 0.3441 - 6(0.303))$

$= -0.5549 \quad -0.5181$

5th. $x^{(5)} = \frac{1}{10}[3 + 5(0.2841) + 2(-0.5549)]$

$= 0.404$

$y^{(5)} = \frac{1}{10}[3 + 4(0.3441) + 3(-0.5549)]$

$z^{(5)} = \frac{1}{10}(-3 - 0.3441 - 6 \times 0.4717)$

$x^{(5)} = 0.34$

$y^{(5)} = 0.28$

$z^{(5)} = -0.50$

$x^{(6)} = 0.34$

$y^{(6)} = 0.28$

$z^{(6)} = -0.50$

Here, 5th & 6th approximation are same.

Hence, $x = 0.34$

$y = 0.28$

$z = -0.50$.

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