Fluid Dynamics

Generally the force acting on fluid elements are pressure force (F_p), Gravity force (F_g) and viscus force ~~F_p~~ (F_v) in Navier-stroke analysis, these three forces are taken into consideration.

In Euler's analysis viscus forces are neglected, therefore only pressure and gravity forces are taken into consideration, in Euler's analysis.

Euler Equation

Assumption

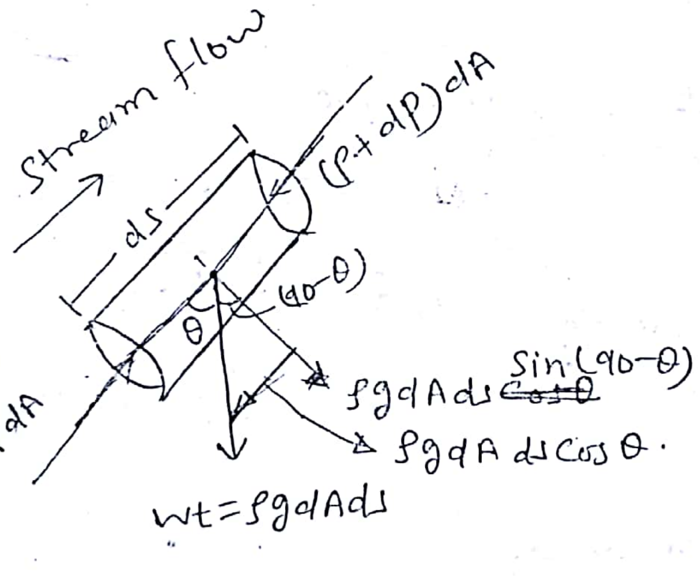
(1) The flow is non- ~~viscus~~ viscous.

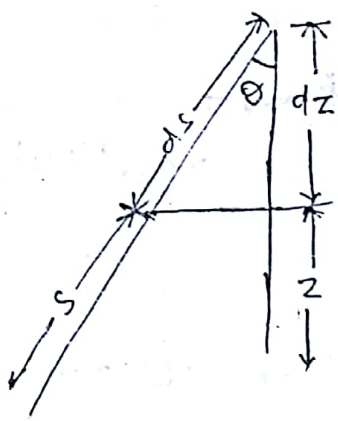
$$W = mg$$

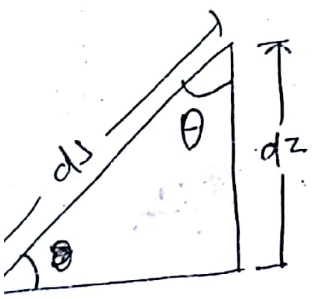
$$m = W/g$$

$$m = \frac{\rho g \, dA \, ds}{g}$$

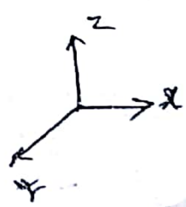
$$\boxed{m = \rho \, dA \, ds}$$







$$\boxed{dz = ds \cos\theta}$$



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$$\boxed{\Sigma F = m a_s}$$

$$\boxed{a_s = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t}}$$

$$P dA - (P + dP) dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \left[\frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]$$

$$-dP - \rho g\, ds \cos\theta = \rho\, ds \left[\frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]$$

$$\boxed{-dP - \rho g\, dZ = \rho\, ds \left[v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]}$$

Euler's Equation.

Bernoulli's equation (Coservation of Energy)

Bernoull's equation is obtain by integrating Euler's equation under Some Conditions.

Assumption

(1) Non-viscus flow

(2) Flow is along stream Line.

(3) No addition or Substraction of energy from the fluid.

(4) steady flow

(5) Incompressible flow.

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$$-dP - \rho g\,dz = \rho\,ds\left[\frac{v\,\partial v}{\partial s} + \cancel{\frac{\partial v}{\partial t}}\right]$$

Steady

$$-dP - \rho g\,dz = \rho\,\cancel{ds}.\,v.\,\frac{dv}{\cancel{ds}}$$

$$-dP - \rho g\,dz = \rho v\,dv$$

$$dP + \rho g\,dz + \rho v\,dv = 0$$

$$\frac{dP}{\rho} + g\,dz + v\,dv = 0$$

Integrate

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{Constant.}$$

For incompressible $\rho = \text{Constant.}$

$$\boxed{\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{Constant.}}$$

Bernoulli's equation.

In the above equation each terms represents energy of the fluid per unit mass.

$$\frac{\text{Energy}}{\text{mass}} \leftarrow \boxed{\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{Constant}}$$

$$\frac{1}{\rho} = v \,, \qquad \frac{P}{\rho} = Pv$$

$$KE = \frac{1}{2} mV^2$$

$$\frac{KE}{m} = \frac{V^2}{2}$$

$$\boxed{w = \rho g}$$

$$\frac{\text{energy}}{mg} \leftarrow \boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}}$$

$$\frac{P}{w} + \frac{V^2}{2g} + Z = \text{Constant.}$$

$\frac{P}{w}$ = Pressure head.

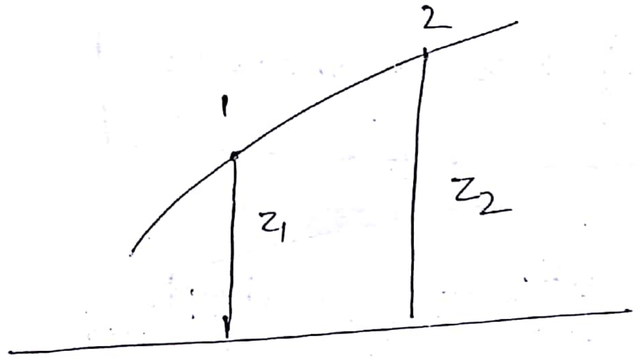
$\frac{V^2}{2g}$ = Kinetic head or Dynamic head.

$Z \rightarrow$ Potential Energy head or datum head.

$$\boxed{\frac{P}{w} + Z = \text{Piezometric head.}}$$

Note :-

Bernoulli's equation is applicable b/w any two points in irrotational flow because the value of constant is same for all stream lines, but whereas for rotational flow Bernoulli's equation must be applied only along stream line because the value of stream line constant is different for different stream lines.

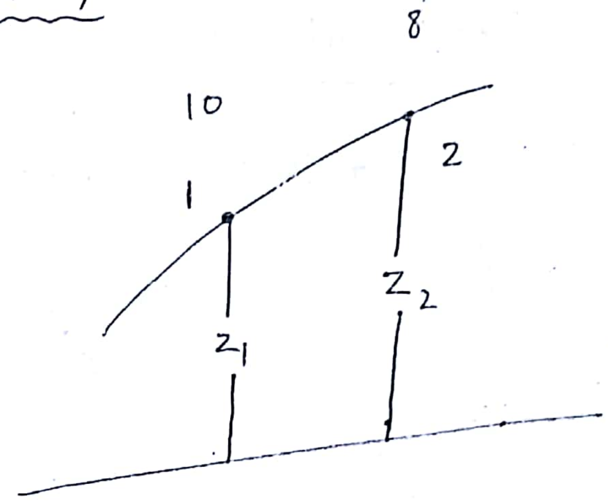


$$\boxed{\frac{P}{w} + \frac{V^2}{2g} + Z = \text{Constant}}$$

Classical Bernoulli's equation.

Bernoulli's equation for a real fluid flow problem

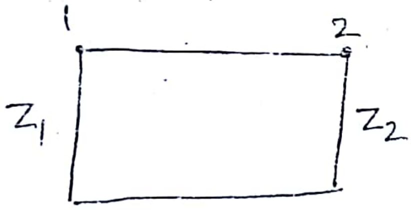
$\boxed{10 = 8 + x}$



$$\boxed{\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 + h_L}$$

Modified B' equation for real fluid.

h_L = Head loss b/w ① & ②



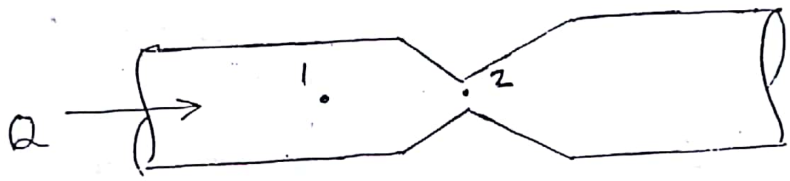
$$\frac{P_1}{w} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{w} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$$\boxed{\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}}$$

Application of Bernoulli's equation

(I) Venturimeter

It is used for measuring discharge through a pipe.



$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g} = h$$ → difference of pressure head at section 1 and 2 and is equal to h.

$$\frac{V_2^2 - V_1^2}{2g} = h$$

$Q = a_1V_1 = a_2V_2$

$V_2 = \frac{Q}{a_2}$, $V_1 = \frac{Q}{a_1}$

$$\frac{Q^2}{a_2^2} - \frac{Q^2}{a_1^2} = 2gh$$

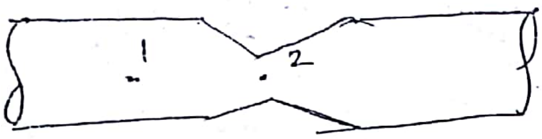
$$Q^2\left[\frac{a_1^2 - a_2^2}{a_1^2 a_2^2}\right] = 2gh$$

$$Q^2 = \frac{a_1^2 a_2^2 \times 2gh}{a_1^2 - a_2^2}$$

$$\boxed{Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

a_1 = Pipe area

a_2 = Throat area.



$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

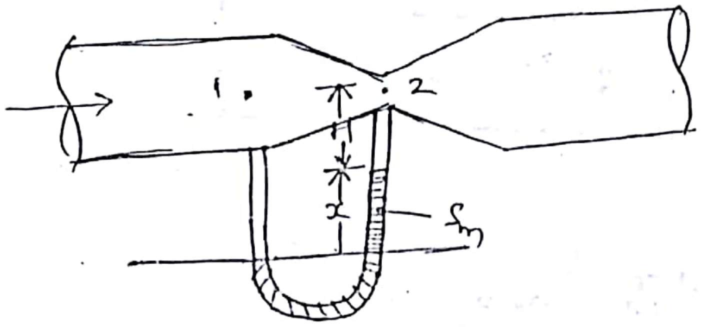
$$a_1 v_1 = a_2 v_2$$

as

$$\boxed{a_2 < a_1}$$

$$V_2 > V_1 \Rightarrow P_2 < P_1$$

$$\boxed{xS_m = h_{air} \times S}$$



$$\frac{P_1}{w} + (\cancel{H}+x) - \frac{xS_m}{S} - \cancel{H} = \frac{P_2}{w}$$

$$\frac{P_1 - P_2}{w} = x\left(\frac{S_m}{S} - 1\right)$$

$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$h = \frac{P_1 - P_2}{w} = x\left(\frac{S_m}{S} - 1\right)$$

$$\boxed{h = x\left(\frac{S_m}{S} - 1\right)}$$

~~or~~ If $S_m < S$

$$\boxed{\frac{P_1}{w} - \frac{P_2}{w} = h = x\left(1 - \frac{S_m}{S}\right)}$$

S_m → manometric liquid

~~of~~ Sp. gravity of manometric liq.

S → Liquid flowing in venturimeter.

Principal of Venturimeter :-

area
By decreasing ↓ in steady one-dimensional incompressible flow velocity increases (from continuity equation) this increase velocity results in decrease in pressure (from Bernoull's eqn, thus their is pressure difference b/w pipe and throat sections, if differential manometer is connect b/w these two points, their is a manometric fluid deflection. By measuring this deflection ∝ discharge can be found.

* While the obtaining the above equation for discharge as no losses are taken into consideration, such a discharge is known as ideal or theoretical discharge.

$$\boxed{Q_{ideal} = Q_{th} = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

Practical application of Bernoull's Equation.

Although Bernoull's equation is applicable in all problems of incompressible flow where there is involvement of energy consideration but ~~there~~ here we shall discuss its applications in the following measuring devices.

1. Venturimeter
2. Orificemeter
3. Pitot tube.

Types of venturimeter

1) Horizontal venturimeter.
2) vertical venturimeter.
3) Inclined venturimeter.

* The value of streamline constant is same for streamline in case of irrotational flow and hence a bernoulli's equation can be applied throughout the flow ~~flow~~ fluid. (Along or across the stream line) for irrotational flow.

In case of rotational flow bernoulli's equation is applied only along stream line (Not across the stream line)

Co-efficient of discharge (C_d)

$$C_d = \frac{Q_{act}}{Q_{th}}$$

$$\boxed{Q_{act} = C_d . Q_{th}.}$$

$$\boxed{Q_{act} = \frac{C_d . a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

As venturimeter gradually converging and diverging device losses are less and C_d is equal to 0.94 to 0.98.

"The value of C_d depends on area ratio and Reynolds number."

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} + h_L$$

$$\frac{P_1 - P_2}{w} - h_L = \frac{V_2^2 - V_1^2}{2g}$$

$$h - h_L = \frac{V_2^2 - V_1^2}{2g}$$

$$\boxed{V_2^2 - V_1^2 = 2g(h - h_L)}$$

$$\boxed{Q_{act} = a_1 V_1 = a_2 V_2}$$

$$V_1 = \frac{Q_{act}}{a_1} \quad , \quad V_2 = \frac{Q_{act}}{a_2}$$

$$\boxed{\frac{Q_{act}^2}{a_2^2} - \frac{Q_{act}^2}{a_1^2} = 2g\,(h - h_L)}$$

$$Q_{act}^2 \left[\frac{a_1^2 - a_2^2}{a_1^2 a_2^2}\right] = 2g(h - h_L)$$

$$Q_{act} = \frac{a_1 a_2 \sqrt{2g(h_1 - h_L)}}{\sqrt{a_1^2 - a_2^2}} \longrightarrow (1)$$

$$Q_{act} = \frac{C_d\, a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$\frac{\cancel{a_1 a_2} \sqrt{\cancel{2g}(h_1 - h_L)}}{\cancel{\sqrt{a_1^2 - a_2^2}}} = \frac{C_d\, \cancel{a_1 a_2} \sqrt{\cancel{2g}\,h}}{\cancel{\sqrt{a_1^2 - a_2^2}}}$$

$$C_d \sqrt{h} = \sqrt{h - h_L}$$

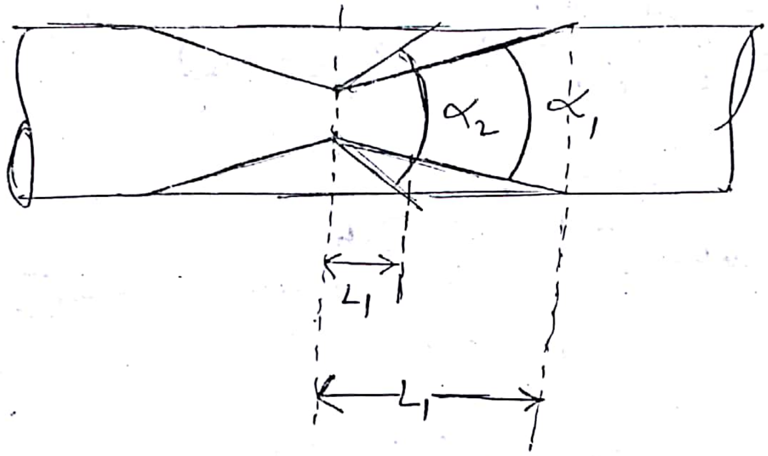
$$\boxed{C_d = \sqrt{\frac{h - h_L}{h}}}$$

General properties of venturimeter.

The throat diameter $d_2 = \left(\frac{1}{3} \text{ to } \frac{1}{2}\right) d_1$

Angle of Convergence = 21° to 22°.

Angle of divergence about 7°.



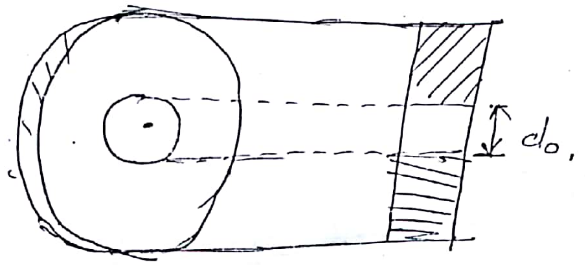
Note: The angle of divergence is kept less 7° in order to avoid boundary layer separation. Therefore to avoid flow separation the diverging length is kept more and diverging angle is kept less.

Orificemeter :-

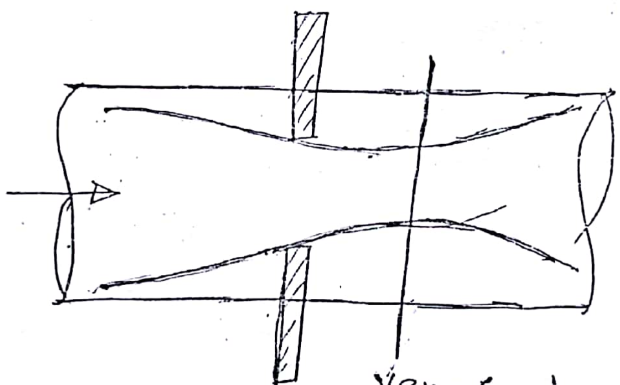
This is used for finding out discharge or flow rate.

This device is based on the same principle of a venturimeter.

It is ~~of~~ the chepest instrument for finding the discharge.



Orificemeter is thin circular tube with a circular hole.



$$\boxed{C_c = \frac{\text{Vena Contracta Area}}{\text{orifice area}}}$$

C_c = Co-efficient of Contraction.

$$Q = \frac{C_d\, a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$C_d =$ 0.68 – 0.76 in orificemeter.

As Area reduction is sudden the losses are more and hence C_d for orifice meter is 0.68 to 0.76.

#

$$C_c = \frac{\text{Vena Contracta area}}{\text{orifice area}}$$

$$C_d = \frac{Q_{act}}{Q_{th}}$$

vena contracta

$$= \frac{A_{act} \cdot V_{act}}{A_{th} \cdot V_{th}}$$

orifice

$$\boxed{C_d = C_c \times C_v}$$

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Orifice meter

$a_0 = A_0 =$ Area of orifice.

Co-efficient of Contraction (C_c)

$$C_c = \frac{\text{Vena Contracta area}}{\text{orifice area}} = \frac{a_2}{a_0}$$

$$\Rightarrow \boxed{a_2 = a_0 \times C_c}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$a_1 v_1 = a_2 v_2$$

$$\frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g} = h$$

$$\boxed{V_2 = \frac{a_1 V_1}{a_2}}$$

$$V_2^2 - V_1^2 = 2gh$$

$$V_1 = \frac{a_2 V_2}{a_1}$$

$$V_1 = \frac{a_0 \times C_c \times V_2}{a_1}$$

$$\cancel{V_2^2 - V_1^2 =}$$

$$V_2^2 - \frac{C_c^2 \times a_0^2 \times V_2^2}{a_1^2} = 2gh$$

$$V_2^2 \left[1 - \frac{C_c^2 \times a_0^2}{a_1^2} \right] = 2gh$$

$$\boxed{V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 \times a_0^2}{a_1^2}}}}$$

$$Q = a_2 V_2$$

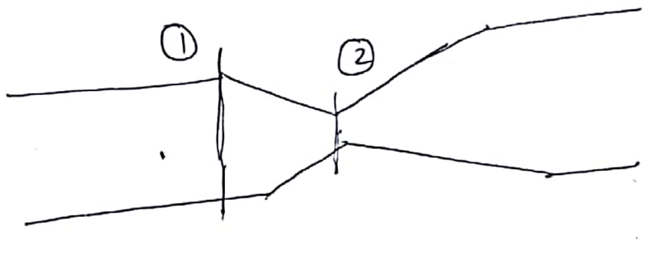
$$Q = \frac{C_c a_0 \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}}$$

$$Q = \frac{C_c \cdot a_0 \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}} \times \frac{\sqrt{1 - a_0^2/a_1^2}}{\sqrt{1 - a_0^2/a_1^2}}$$

$$Q = C_d$$

Problem ⓘ A horizontal venturimeter with inlet diameter 200 mm and throat diameter 100 mm. is used to measure the flow of water. The pressure at inlet is 0.18 N/mm² and the vacuum pressure at the throat is 280 mm of mercury. Find the rate of flow. The value of Cd may be taken as 0.98.

Solution:



$d_1 = 200\,mm = 0.2\,m.$

$a_1 = \frac{\pi}{4}(0.2)^2 = 0.0314\,m^2.$

$d_2 = 100\,mm = 0.1\,m.$

$a_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\,m^2.$

$P_1 = 0.18\,N/mm^2 = 0.18 \times 10^6\,N/m^2.$

$= 180\,KN/m^2$

$$\boxed{\frac{P_1}{w} = \frac{180}{9.81} = 18.3\,m.}$$

Vacuum pressure at the throat

$\frac{P_2}{w} = -280\,mm$ of mercury.

$= -0.28\,m$ of mercury

$= -0.28 \times 13.6 = -3.8\,m$ of water.

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Co-efficient of discharge $\boxed{C_d = 0.98}$

$\therefore$ Differential head $\boxed{h = \frac{P_1}{w} - \frac{P_2}{w}}$

$= 18.3 - (-3.8)$

$= 22.1\ m.$

Rate of flow Q

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$Q = 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - (0.00785)^2}} \times \sqrt{2 \times g \times 22.1}$$

$\boxed{Q = 0.165\ m^3/s}$ Ans

Problem (2) A horizontal venturimeter with inlet diameter 200mm and throat diameter 100mm is employed to measure the flow of water. The reading of the differential manometer connected to the inlet is 180 mm of mercury. If the co-efficient of discharge is 0.98 determine the rate of flow.

Solution:

$d_1 = 200mm = 0.2m.$

$a_1 = \frac{\pi}{4}(0.2)^2 = 0.0314 m^2.$

$d_2 = 100 mm = 0.1 m.$

$a_2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785 m^2.$

Reading of differential manometer $x = 180 mm$
$= (0.18 m)$ of Hg.

$C_d = 0.98.$

Rate of flow (Q).

$h = x\left(\frac{S_m}{S} - 1\right)$ S_m = Sp. gr. of Hg
S = Sp. gr. of flowing liquid flowing through pipe.

$h = 0.18\left(\frac{13.6}{1} - 1\right) = 2.268 m.$

$$\boxed{Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$ we get

$$Q = \frac{0.98 \times 0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 2.268}$$

$$\boxed{Q = 0.0528 \ m^3/s}$$

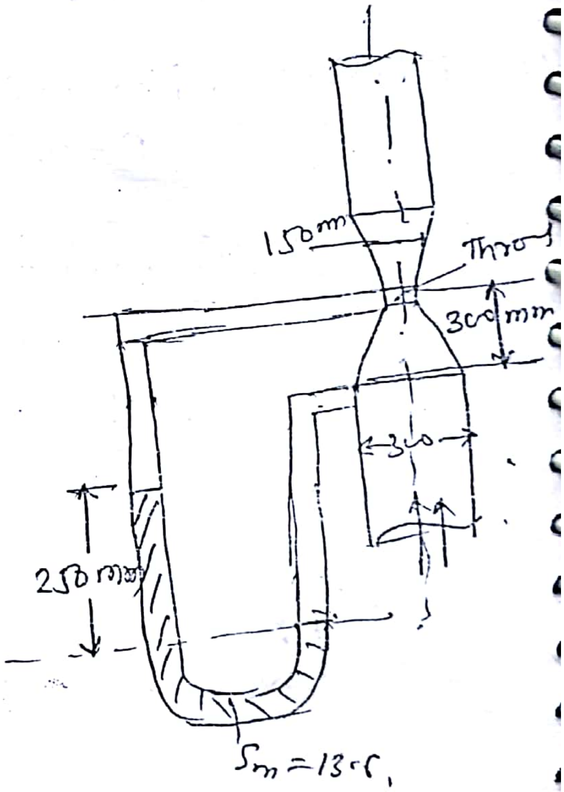
Problem (3) A 300 mm x 150 mm venturimeter is provided in a vertical pipeline carrying oil of sp. gravity 0.9 flow being upward. The difference in elevation of the throat section and entrance section of the venturimeter is 300, The differential U-tube mercury manometer shows a gauge deflection of 250 mm. Calculate

(i) The discharge of oil, and

(ii) The pressure difference b/w entrance section and throat section.

Take the co-efficient of meter as 0.98 and sp. gravity of mercury as 13.6.

Solution



$d_1 = 300\,mm = 0.3\,m.$

$a_1 = \frac{\pi}{4} \times 0.3^2 = 0.0706\ m^2.$

$d_2 = 150\,mm = 0.15\,m.$

$a_2 = \frac{\pi}{4} \times 0.15^2 = 0.01766\ m^2.$

$S_m = 13.6,$

$S_l = 0.9$

Reading of differential manometer $x = 250\ mm.$
$= 0.25\,m.$

$$h = \left(\frac{P_1}{w} + z_1\right) - \left(\frac{P_2}{w} + z_2\right)$$

$$= x\left[\frac{S_m}{S} - 1\right] = 0.25\left[\frac{13.6}{0.9} - 1\right] = 3.53\ m \text{ of oil.}$$

i) Discharge of oil, Q

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2 \times g \times h}$$

$$Q = 0.98 \times \frac{0.07 \times 0.01767}{\sqrt{0.07^2 - 0.01767^2}} \times \sqrt{2 \times 9.81 \times 3.53}$$

$$\boxed{Q = 0.1489 \ m^3/s.}$$ Ans.

) Pressure difference b/w entrance and throat section $P_1 - P_2$

$$h = \left(\frac{P_1}{w} + z_1\right) - \left(\frac{P_2}{w} + z_2\right) = 3.53.$$

$$\left(\frac{P_1}{w} - \frac{P_2}{w}\right) + (z_1 - z_2) = 3.53$$

$Z_2 - Z_1 = 300$ mm or 0.3 m.

$$\therefore \left(\frac{P_1}{w} - \frac{P_2}{w}\right) - 0.3 = 3.53.$$

$$\frac{P_1 - P_2}{w} = 3.83$$

$$P_1 - P_2 = ([0.9 \times 10^3) \times 9.81] \times 3.83$$

$$= 33815.02 \ N/m^2$$

$$= 33.815 \ KN/m^2 \longrightarrow$$ Ans

Introduction:

Flow through orifices and mouthpieces

Flow measuring devices namely venturimeter, orificemeter, working principle of those based on the application of Bernoulli's equation.

Orifices and mouthpieces are also commonly used for the measurement of flow rate.

An orifice is a small opening usually round on the side or at the bottom of a tank, through which the fluid flows.

Classification of orifices

1) According to size.

(i) Small orifices (ii) Large orifices.

(2) According to shape

(i) circular orifice (ii) Rectangular orifices.
(iii) Triangular orifice (iv) square orifice.

(3) Shape of upstream edge

(i) Sharp-edged orifice (ii) Bell-mouthed orifice.

(4) According to discharge Conditions.

(i) Free discharge orifices
(ii) Drowned or submerged orifices.
(iii) Fully submerged
(iv) partially submerged.

Hydraulic Co-efficient

1) Co-efficient of velocity C_v

2) Co-efficient of Contraction C_c

3) Co-efficient of discharge. C_d.

(1) Co-efficient of velocity (C_v) ⇒ It is defined as the ratio b/w the actual velocity of a jet of liquid at vena-contracta and theoretical velocity of jet, It is denoted by C_v.

$$C_v = \frac{\text{Actual velocity of jet at vena-Contracta}}{\text{Theoretical velocity.}}$$

$$= \frac{V}{\sqrt{2gH}}$$ where V = actual velocity,

$\sqrt{2gH}$ = Theoretical velocity.

The value of C_v varies from 0.95 to 0.99 for the different orifices, depending on the shape, size of the orifices and on the head under which flow takes place.

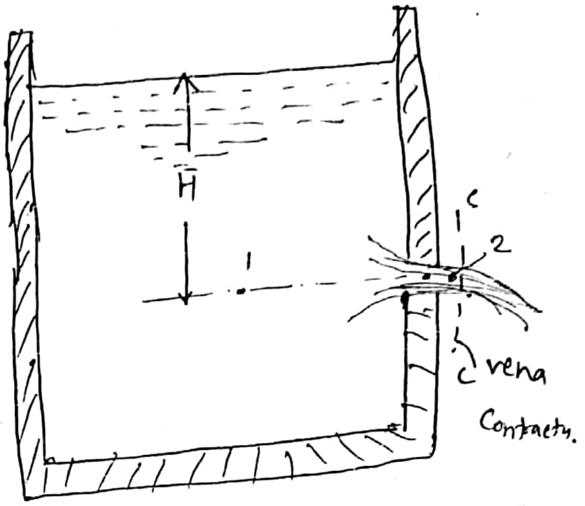
Generally the value of $C_v = 0.98$ is taken for Sharp-edged orifices.

ORIFICES AND MOU

* Flow Through an orifices

Let

H = head of the liquid above the centre of the orifices



Tank with an orifices.

Consider two point 1 and 2.

Point 1 is inside the tank and point 2 at the vena-contracta.

Let the flow is steady and at a constant head.

Applying Bernoulli's equation at point 1 and 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\cancel{H + \frac{V_1^2}{2g}}$$

V_1 is very small in comparison to V_2 as area of ~~take~~ tank is very large as compared to the area of the jet of liquid.

$P_1 = \rho g H$

$H = \frac{P_1}{\rho g}$

$\frac{P_2}{\rho g} = 0 \rightarrow$ atmospheric pressure.

$$H + 0 = 0 + \frac{V_2^2}{2g}$$

$$\boxed{V_2 = \sqrt{2gH}}$$

This is theoretical velocity. Actual velocity will less than this value.

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Co-efficient of Contraction (C_c)

It is defined as the ratio of the area of the jet at vena-Contracta to the area of the orifice. It is denoted by C_c.

Let a = area of orifice

a_c = area of jet at vena-Contracta,

$$C_c = \frac{\text{area of jet at vena-Contracta}}{\text{area of orifice}}$$

$$\boxed{C_c = \frac{a_c}{a}}$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may taken as 0.64.

Co-efficient of Discharge (C_d) :- It is ~~denoted~~ defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically C_d is given as

$$C_d = \frac{Q_a}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Th. velocity} \times \text{Th. area}}$$

$$\boxed{C_d = C_v \times C_c}$$

The value of C_d varies from 0.61 to 0.65. For general purpose the value of C_d is taken as 0.62.

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Prob The head of water over an orifice of diameter 40mm is 10m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$.

Solution

$$H = 10\ cm$$

$$d = 40mm = 0.04m.$$

$$a = \frac{\pi}{4}(0.04)^2 = 0.001256m^2.$$

$$C_d = 0.6$$

$$C_v = 0.98$$

(i)

$$\frac{\text{Actual discharge}}{\text{Theoretical discharge}} = C_d = 0.6$$

Th. discharge = V_{th} x area of orifice.

$$V_{th} = \sqrt{2gH}$$

$$= \sqrt{2 \times 9.81 \times 10} = 14m/s.$$

$$Q_{th} = 14 \times 0.001256 = 0.01758\ m^3/s.$$

$$Q_{act} = 0.6 \times Q_{th}$$

$$= 0.6 \times 0.01758$$

$$= 0.01054\ m^3/s \rightarrow Ans$$

Orifice meter

d_1 = Pipe diameter $\longrightarrow a_1$

$d_2 \Rightarrow$ dia of vena contracta $\longrightarrow a_2$

d_0 = dia of orifice $\longrightarrow a_0$

$$C_c = \frac{a_2}{a_0}$$

$$\boxed{a_2 = C_c a_0}$$

$$\frac{P_1}{w} + \frac{v_1^2}{2g} = \frac{P_2}{w} + \frac{v_2^2}{2g}$$

$$\frac{P_1 - P_2}{w} = \frac{v_2^2 - v_1^2}{2g} = h$$

$$\boxed{v_2^2 - v_1^2 = 2gh}$$

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

$$v_1 = \frac{C_c \times a_0 v_2}{a_1}$$

$$v_2^2 - \frac{C_c^2 \times a_0^2 \times v_2^2}{a_1^2} = 2gh$$

$$v_2^2\left[1 - \frac{C_c^2 a_0^2}{a_1^2}\right] = 2gh$$

$$\boxed{V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}}}$$

$$Q = a_2 v_2$$

$$\boxed{Q = \frac{C_c \times a_0 \times \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}}}$$

$$Q = \frac{C_c \times a_0 \times \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1}}} \times \frac{\sqrt{1 - \frac{a_0^2}{a_1^2}}}{\sqrt{1 - \frac{a_0^2}{a_1}}}$$

$$\boxed{C = C_d = \frac{C_c \times \sqrt{1 - \frac{a_0^2}{a_1}}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1}}}}$$

$$\boxed{Q = \frac{C_d\, a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

$C \to$ Constant $\Rightarrow$ orifice meter constant & also called C_d.

Problem: An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauge fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.

Solution:

$d_0 = 10$ cm.

$a_0 = \frac{\pi}{4}(10)^2 = \boxed{78.54\ cm^2}$

$d_1 = 20$ cm.

$a_1 = \frac{\pi}{4} \times (20)^2 = \boxed{314\ cm^2}$

$P_1 = 19.62\ N/cm^2$

$= 19.62 \times 10^4\ N/m^2$

$\frac{P_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20$ m of water.

$\boxed{\begin{array}{l} 1\ \text{litre} = \frac{1}{1000} m^3 \\ 1\ \text{litre} = 1000\ cm^3 \end{array}}$

$\frac{P_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10$ m of water.

$h = \frac{P_1}{w} - \frac{P_2}{w} = 20 - 10 = 10$ m of water
$= 1000$ cm of water.

$\boxed{C_d = 0.6}$

$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$

$Q = \frac{0.6 \times 78.54 \times 314}{\sqrt{\cancel{78.54^2 -} 314^2 - 78.54^2}} \times \sqrt{2 \times \cancel{9.81}\,981 \times 1000}$

$\boxed{Q = 68174.3\ cm^3/s}$

$\boxed{Q = 68.17\ \text{litres/s.}}$

Problem An orifice meter with orifice diameter 15cm is inserted in a pipe of 30cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp-gr. 0.9 when the Co-efficient of discharge of the orifice meter = 0.64.

Solution :-

$d_0 = 15\,cm.$ $\quad a_0 = \frac{\pi}{4}(15)^2 = 176.62\,cm^2.$

$d_1 = 30\,cm$ $\quad a_1 = \frac{\pi}{4}(30)^2 = 706.5\,cm^2.$

$x = 50\,cm$ of Hg.

$$h = x\left(\frac{S_m}{S_o} - 1\right) = 50 \times \left[\frac{13.6}{0.9} - 1\right] \text{ cm of oil.}$$

$$= 705.5 \text{ cm of oil.}$$

$$\boxed{C_d = 0.64}$$

The rate of flow, Q

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$Q = 0.64 \times \frac{176.62 \times 706.5}{\sqrt{706.5^2 - (176.62)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$Q = 137350.86\ cm^3/s.$

$= 137.35$ liter/sec.

Problem: A horizontal venturimeter with inlet and throat diameters 300 mm and 100 mm respectively is used to measure the flow of water. The pressure intensity at inlet is 130 kN/m² while the vacuum pressure head at the throat is 350 mm of mercury. Assuming that 3 Percent of head is lost in b/w the inlet and throat, find:

(i) The value of C_d (Co-efficient of discharge) for the the venturimeter and

(ii) Rate of flow.

Solution: $d_1 = 300\,mm = 0.3\,m$. $\quad a_1 = \frac{\pi}{4}(0.3)^2$ $\quad \boxed{a_1 = 0.07\,m^2}$

$d_2 = 100\,mm = 0.1\,m \Rightarrow a_2 = \frac{\pi}{4}(0.1)^2$ $\quad \boxed{a_2 = 0.00785\,m^2}$

$P_1 = 130\,kN/m^2$

$\therefore$ pressure head, $\frac{P_1}{w} = \frac{130}{9.81} = \boxed{13.25\,m.}$

Similarly pressure head at throat.

$$\frac{P_2}{w} = -350\,mm \text{ of } Hg$$
$$= -0.35 \times 13.6 \text{ m of water} = \boxed{-4.76\,m}$$

(i) Co-efficient of discharge.

$$h = \frac{P_1}{w} - \frac{P_2}{w} = 13.25 - (-4.76) = 18.01\,m.$$

Head lost $\quad h_f = 3\%$ of $h = \frac{3}{100} \times 18.01 = 0.54\,m.$

$$C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{18.01 - 0.54}{18.01}} = \boxed{0.985}$$

ii) Rate of flow Q

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

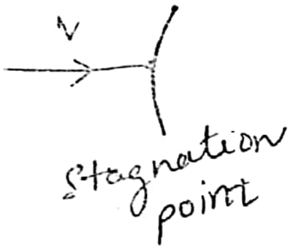
$$Q = 0.985 \times \frac{0.07 \times 0.00785}{\sqrt{0.07^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 18.01}$$

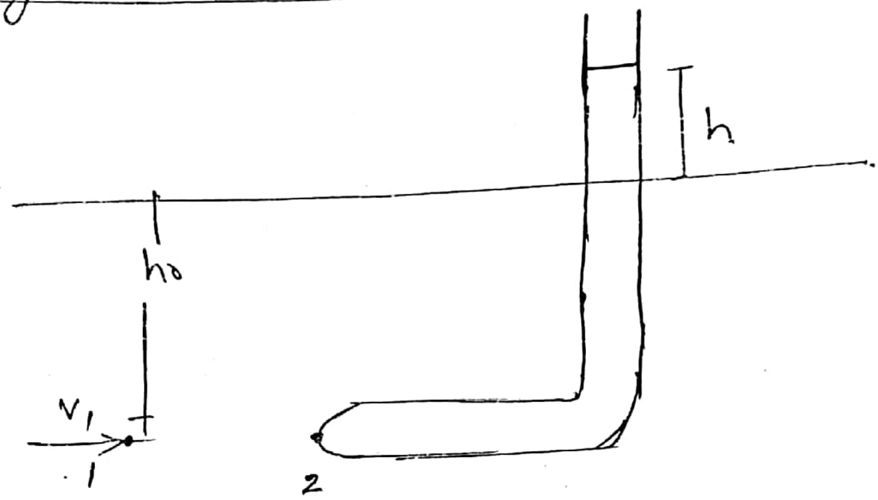
$$\boxed{Q = 0.146 \ m^3/s}$$

Pitot tube :

This is device used for finding velocity of flow.

Case - I   Velocity in open channel.





$$P_1 = \rho g h$$

$$\frac{P_1}{\rho g} = h_0 \Rightarrow \boxed{\frac{P_1}{w} = h_0}$$

$$\frac{P_1}{w} + \frac{v_1^2}{2g} = \frac{P_2}{w} + \frac{v_2^2}{2g} \quad - \; v_2 = 0$$

(stagnation point)

$$\boxed{\frac{P_1}{w} + \frac{v_1^2}{2g} = \frac{P_2}{w}}$$

static head   Dynamic head.   stagnation head.

$$\rho g h + \rho g h_0 = P_2$$

$$\boxed{\frac{P_2}{\rho g} = h + h_0}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$z_1 = z_2$$

$$\frac{P_1}{w} = h_0 \quad , \frac{P_2}{w} = h + h_0$$

$$\cancel{h_0} + \frac{V_1^2}{2g} = h + \cancel{h_0}$$

$$\frac{V_1^2}{2g} = h \rightarrow \text{Dynamic head}$$

$$\boxed{V_1 = \sqrt{2gh}}$$

$$V = \sqrt{2g\,(\text{Dynamic head})}$$

$$\boxed{V = \sqrt{2g\left(\text{Stagnation head} - \text{Static head}\right)}}$$

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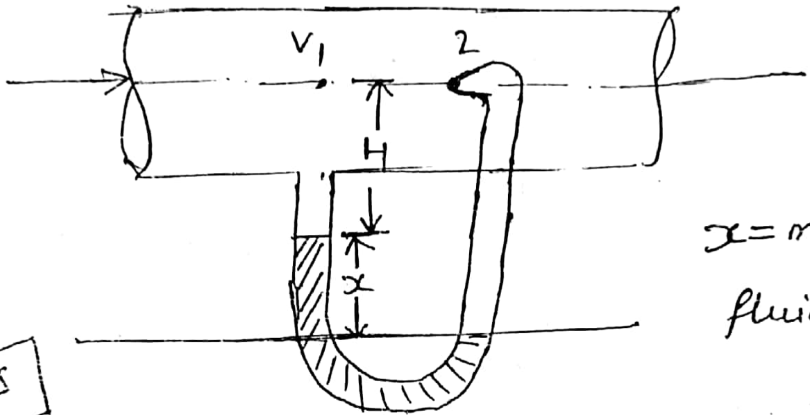
Velocity in pipes

Stagnation point: → It is the point at which the fluid is brought to rest isentropically.



x = manometric fluid deflection.

$\boxed{x S_m = h_{our} \times S}$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}}$$ $V_2 = 0$ (stagnation point).

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w}$$

$$\boxed{\frac{V_1^2}{2g} = \frac{P_2 - P_1}{w}} \longrightarrow (1)$$

$$\frac{P_1}{w} + H + \frac{x S_m}{S} - x - H = \frac{P_2}{w}$$

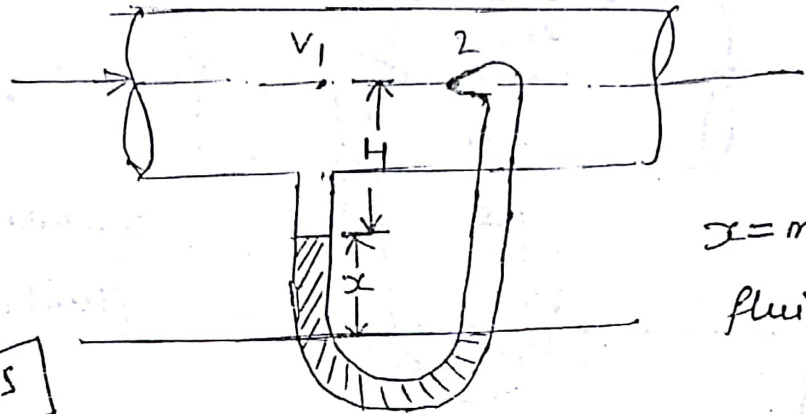
$$\boxed{\frac{P_2 - P_1}{w} = x\left(\frac{S_m}{S} - 1\right)}$$

From (1) & (2)

$$\frac{V_1^2}{2g} = x\left(\frac{S_m}{S} - 1\right) \Rightarrow \boxed{V_1 = \sqrt{2gx\left(\frac{S_m}{S} - 1\right)}}$$

Velocity in pipes

Stagnation point : → It is the point at which the fluid is brought to rest isentropically.



x = manometric fluid deflection.

$\boxed{x S_m = h_{our} \times S}$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} \qquad V_2 = 0 \text{ (stagnation point).}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w}$$

$$\boxed{\frac{V_1^2}{2g} = \frac{P_2 - P_1}{w}} \longrightarrow (1)$$

$$\frac{P_1}{w} + H + \frac{x S_m}{S} - x - H = \frac{P_2}{w}$$

$$\boxed{\frac{P_2 - P_1}{w} = x\left(\frac{S_m}{S} - 1\right)}$$

From (1) & (2)

$$\frac{V_1^2}{2g} = x\left(\frac{S_m}{S} - 1\right) \Rightarrow \boxed{V_1 = \sqrt{2gx\left(\frac{S_m}{S} - 1\right)}}$$

#

$$\boxed{V_{act} = C_v\sqrt{2gh}}$$

Problem. A sub-marine moves horizontally in sea and has it axis 15 m below the surface of water, A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two-limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of freshwater.

Solution :-

$x = 170\,mm = 0.17\,m.$

$S_m = 13.6$

$S = 1.026$

$$h = x\left[\frac{S_m}{S} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right]$$

$$= 2.08\,m.$$

$$\boxed{V = \sqrt{2gh}}$$

$$\Rightarrow V = \sqrt{2 \times 9.81 \times 2.08} = 6.388\ m/s. \rightarrow Ans$$

Problem: Petroleum oil (sp.gr = 0.93) and viscosity = 13 cp) flows isothermally through a horizontal 5cm pipe. A pitot tube is inserted at the centre of pipe and its leads are filled the leads are filled with the same oil and attached to a u-tube containing water. The reading on the manometer is 10 cm. Calculate the volumetric flow of oil in m^3/s. The Co-efficient of pitot tube is 0.98.

Solution: sp. gr. of oil = 0.9,

$$\mu = 13\ cp = \frac{13}{100} \times 0.1 \frac{Ns}{m^2} = 0.013\ Ns/m^2$$

$$x = 10\ cm\ of\ hg = 0.1\ m\ of\ hg$$

$$D = 5\ cm = 0.05\ m.$$

$$C_v = 0.9.$$

Volumetric flow of oil

$$h = x\left(\frac{S_{hg}}{S_{ol}} - 1\right) = 0.1\left(\frac{13.6}{0.9} - 1\right)$$
$$= 1.411.$$

∴ Actual velocity of flow

$$V = C_v\sqrt{2gh}$$
$$= 0.98\sqrt{2 \times 9.81 \times 1.411}$$
$$= 5.156\ m/s$$

Volumetric flow of oil = A × V

$$= \frac{\pi}{4} \times 0.05^2 \times 5.156$$
$$= 0.01\ m^3/s. \rightarrow Ans.$$

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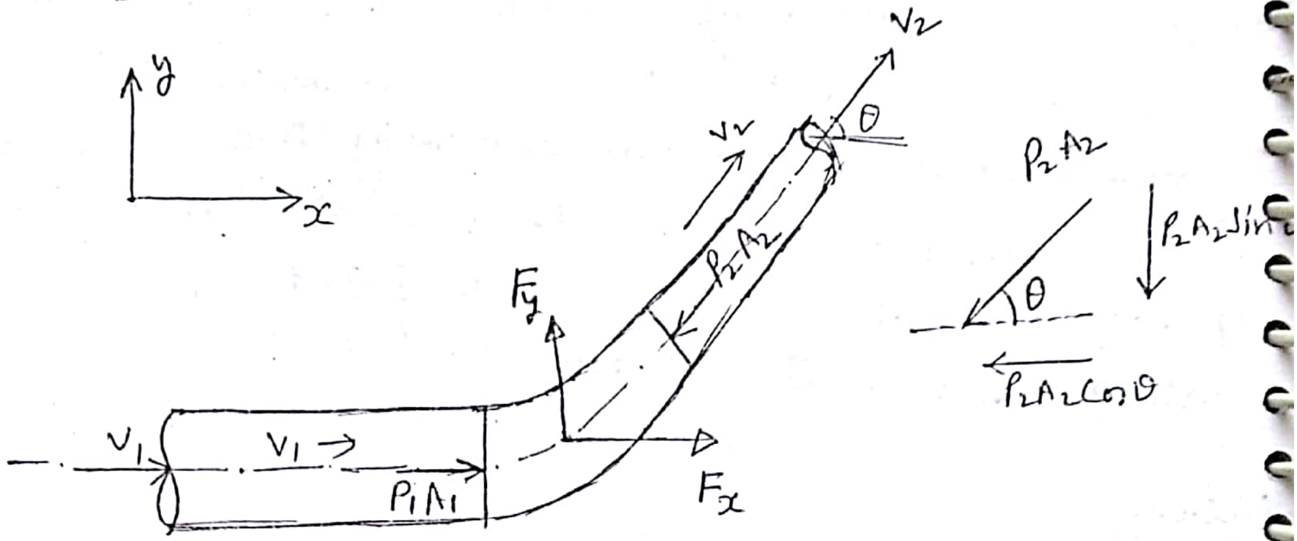
Force on pipe bent



Mometum eqn:

$$\Sigma F = ma.$$

$$\Sigma F = m \times \left(\frac{v-u}{t}\right)$$

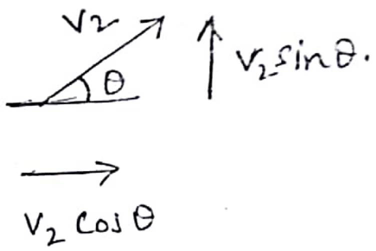
$$\Sigma F = \dot{m}(v-u)$$

$$\dot{m} = \rho AV$$

$$\boxed{AV = Q}$$

$$\dot{m} = \rho Q$$

$$\boxed{\Sigma F = \rho Q(v-u)}$$



momentum eqn

$$\Sigma F = ma$$

$$\Sigma F = \frac{m(v-u)}{t} = \dot{m}(v-u)$$

$$\dot{m} = \rho AV = \rho Q$$

$$\boxed{\Sigma F = \rho Q(V-u)}$$

momentum equation in x-direction.

$$\boxed{P_1A_1 + F_x - P_2A_2\cos\theta = \rho Q(V_2\cos\theta - V_1)}$$

momentum equation in y-direction.

$$F_y - P_2A_2\sin\theta = \rho Q[V_2\sin\theta - 0]$$

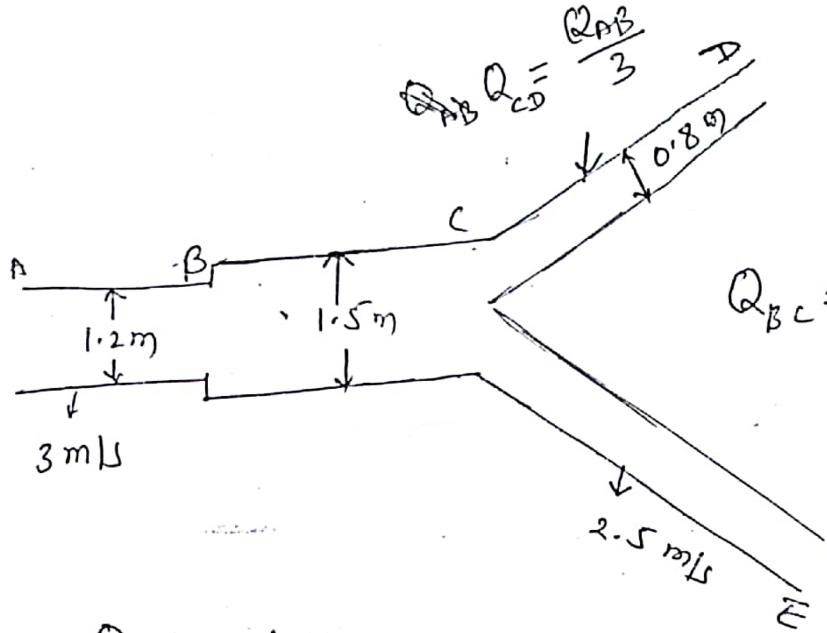
$$\boxed{F_y - P_2A_2\sin\theta = \rho Q V_2\sin\theta.}$$

blem:- A 45° reducing bend is connected in a pipe line, the dimeters at the inlet and outlet of the bend being 600mm and 300mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/sec.

lution:

Problem: Water flows through a pipe AB 1.2 m diameter at 3 m/s and then passes through a pipe BC = 1.5 m diameter. At C, the pipe branches. Branch CD is 0.8 m in diameter and carries one-third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

Solution



$$Q_{BC} = Q_{CD} + Q_{CE}$$

$$Q_{AB} = A \times V$$

$$= \frac{\pi}{4} \times (1.2)^2 \times 3 = 3.3912 \text{ m}^3/\text{s}. \quad \text{Ans.}$$

$$A_{AB} V_{AB} = A_{BC} V_{BC}$$

$$V_{BC} = \frac{\frac{\pi}{4} \times (1.2)^2 \times 3}{\frac{\pi}{4} \times (1.5)^2} = 1.92 \text{ m/s}. \quad \text{Ans.}$$

$$Q_{BC} = A_{BC} \times V_{BC} = \frac{\pi}{4}(1.5)^2 \times 1.92 = 3.3912 \text{ m}^3/\text{s}.$$

$$Q_{CD} = \frac{Q_{AB}}{3}$$

$$Q_{CD} = \frac{3.3912}{3} = 1.13 \ m^3/s.$$

$$Q_{CD} = A_{CD} \times V_{CD}$$

$$1.13 = \frac{\pi}{4}(0.8)^2 \times V_{CD}$$

$$\boxed{V_{CD} = \cancel{1.765} \ 2.249 \ m/s.}$$ Ans

$$Q_{BC} = Q_{CE} + Q_{CD}$$

$$3.3912 = Q_{CE} + 1.13$$

$$\boxed{Q_{CE} = 2.26 \ m^3/s}$$

$$Q_{CE} = A_{CE} \times V_{CE}$$

$$2.26 = \frac{\pi}{4}(d_{CE})^2 \times \cancel{V_{CE}} \ 2.5$$

$$\boxed{d_{CE} = 1.07 \ m}$$ Ans.

The momentum Equation

It is based on the law of Conservation of momentum or on the momentum principle.

Which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

momentum eqn (From Newton's Second law of motion).

$$\cancel{\Sigma} F = ma.$$

$$\cancel{\Sigma} F = m \times \frac{dv}{dt} \qquad a = \frac{dv}{dt}$$

$$\cancel{\Sigma} F = \frac{d(mv)}{dt}$$

$$\boxed{F = \frac{d(mv)}{dt}}$$

This equation is is known as momentum principle.

$$\boxed{F.dt = d(mv)}$$

which is known as the impulse-momentum equation and states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to change of momentum d(mv) in the direction of force.

#

Momentum eqn

$$\Sigma F = ma.$$

$$\Sigma F = \frac{m(v-u)}{t}.$$

$$\boxed{\Sigma F = \dot{m}(v-u)}$$

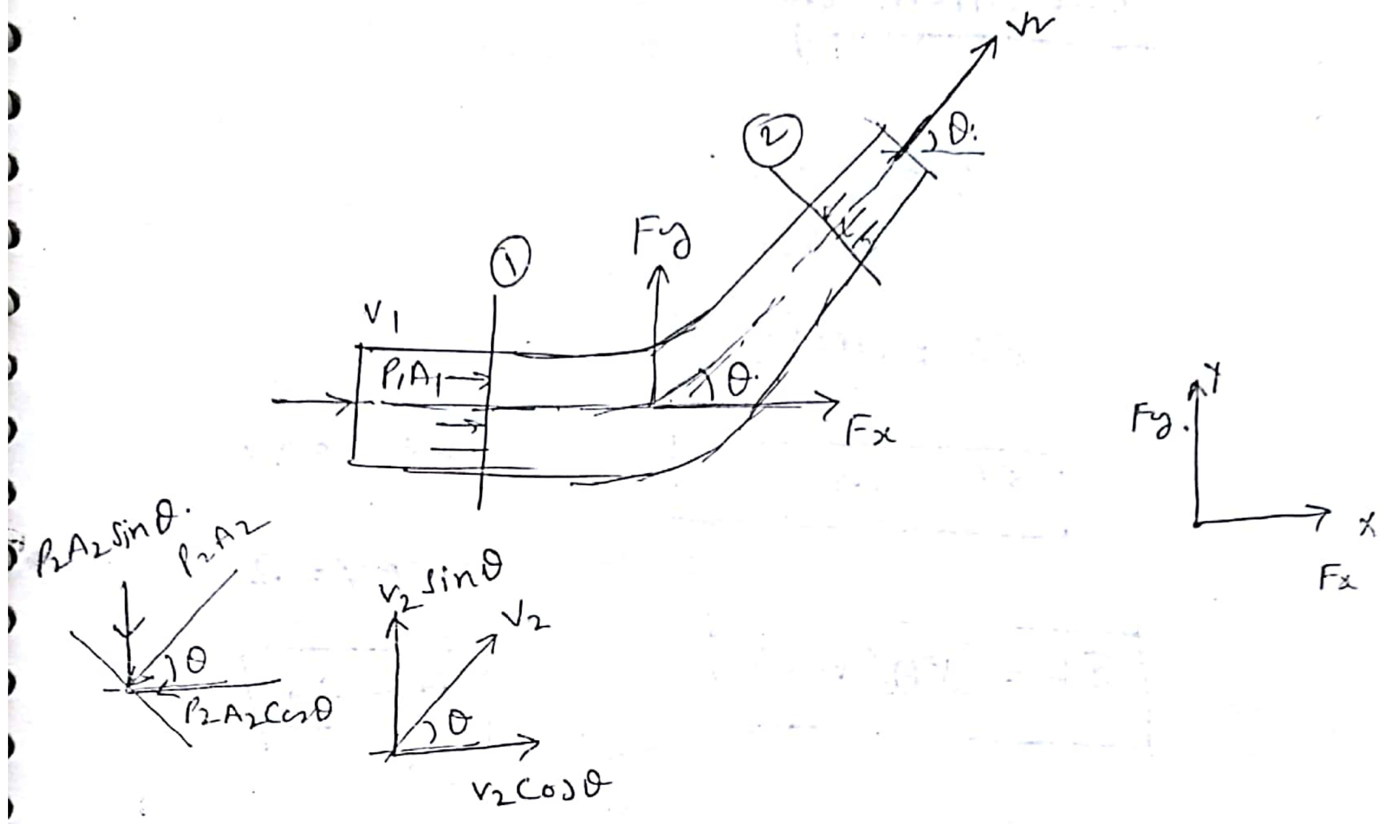
$$\dot{m} = \rho AV.$$

$$AV = Q.$$

$$\boxed{\Sigma F = \rho Q(v-u)}$$

$$\boxed{\dot{m} = \rho Q}$$

Force exerted by a flowing fluid on a pipe-bend.



The force exerted by the bend on the fluid in the directions of x and y will be equal to F_x and F_y but in the opposite directions. Hence component of the force exerted by bend on the fluid in x-direction $= -F_x$ and the direction of $y = -F_y$.

Net force acting on the fluid in the direction of x = Rate of change of momentum in x-direction.

$P_1A_1 - P_2A_2\cos\theta - F_x$ = (mass per sec) (Change in velocity)

$= \rho Q$ (Final velocity in the direction of x − Initial velocity in the direction of x)

$$= \rho Q (v_2 \cos\theta - v_1)$$

$$\boxed{F_x = \rho Q (v_1 - v_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta}$$

Similarly the momentum equation in y-direction.

$$0 - P_2 A_2 \sin\theta - F_y = \rho Q (v_2 \sin\theta - 0)$$

$$\boxed{F_y = \rho Q (-v_2 \sin\theta) - P_2 A_2 \sin\theta}$$

Now the resutant force (F_R) acting on the bend.

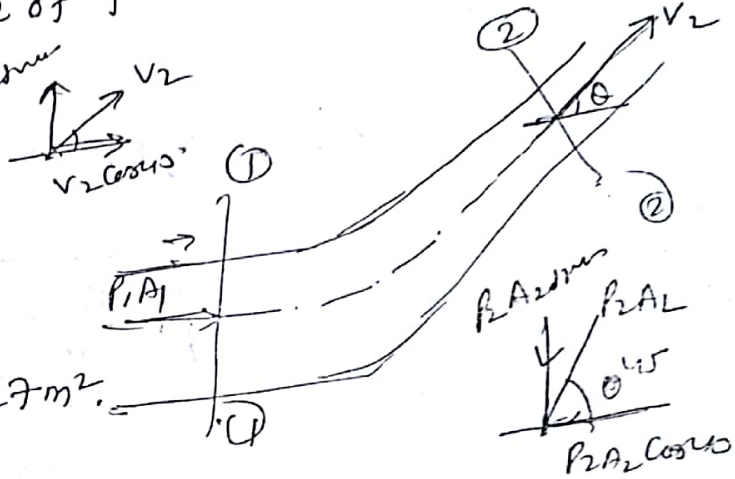
$$= \sqrt{F_x^2 + F_y^2}$$

And the angle made by the ~~resultantant~~ resultant with horizontal direction is given by

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

Problem: A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is $8.829\ N/cm^2$ and rate of flow of water is 600 litres/s.

Solution



$\theta = 45^\circ$

$D_1 = 600mm = 0.6\ m$

$A_1 = \frac{\pi}{4}(0.6)^2 = 0.2827 m^2.$

$D_2 = 300\ mm = 0.3m.$

$A_2 = \frac{\pi}{4}(0.3)^2 = 0.07068\ m^2.$

$P_1 = 8.829 \frac{N}{cm^2} = 8.829 \times 10^4\ N/m^2.$

$Q = 600 \frac{litres}{s} = 0.6\ m^3/s.$

$Q = A_1V_1 \Rightarrow V_1 = \frac{Q_1}{A_1}$

$V_1 = \frac{0.6}{0.2827} = 2.122 m/s.$

$V_2 = \frac{0.6}{0.07068} = 8.488 m/s$

Applying Bernoulli's eq^n

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{P_2}{\rho g} + \frac{8.488^2}{2 \times 9.81}$$

$$\boxed{P_2 = 54518.37\ N/m^2}$$

Forces on the bend in x- and y- directions are given.

$\cancel{F_x = X}$

$\cancel{P_1A_1}$

$$P_1A_1 - P_2A_2\cos45 - F_x = \rho Q(V_2\cos45 - V_1)$$

$$F_x = \rho Q(V_2\cos45 - V_1) - P_1A_1 + P_2A_2\cos45$$

$$F_x = 1000\times0.6\,(8.488\cos45 - 2.122) - 8.829\times10^4\times0.2827$$

$$+ 54518.37\times0.07068\cos45$$

$$\boxed{F_x = 19906.89\ N.}$$

$$-F_y \cancel{F_y} - P_2A_2\sin45 = \rho Q(+V_2\sin45 - 0)$$

$$-F_y = \rho Q[+V_2\sin45] + P_2A_2\sin45$$

$$F_y = \rho Q[(-V_2\sin45)] - P_2A_2\sin45$$

$$F_y = 1000\times0.6\,[-8.488\sin45°] - 54518.37\times0.07068\times\sin45$$

$$\boxed{F_y = -6325.88\ N}$$

−ve sign mean F_y is acting in the downward direction.

$$F_R = \sqrt{19906.89^2 + 6325.88^2} = 20887.77\ N.$$

$$\tan\theta = \frac{F_y}{F_x} = \frac{6325.88}{19906.89} = 17.62°$$

$$= 17°37'$$

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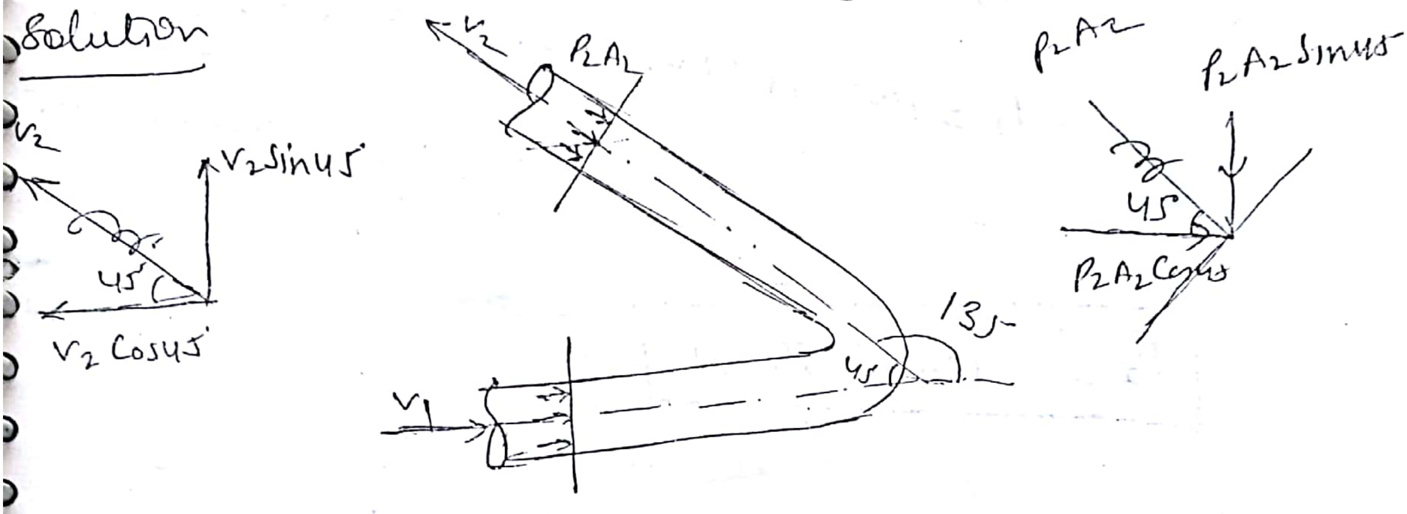
Problem: 250 litres/s of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135° (that is change from initial to final direction is 135°), Find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm².

Solution



$$P_1 A_1 + P_2 A_2 \cos 45 - F_x = \rho Q_2 (-V_2 \cos 45 - V_1)$$

$Q = 250$ litre/s $= 0.25\, m^3/s$.

$D_1 = D_2 = 300\,mm = 0.3\,m \rightarrow A_1 = A_2 = \frac{\pi}{4}(0.3)^2$

$= 0.07068\, m^2$

$P_1 = P_2 = 39.24\, N/cm^2$

$= 39.24 \times 10^4\, N/m^2$.

$V_2 = V_1 = \frac{Q_1}{A_1} = \frac{0.25}{0.07068} = 3.537\, m/s$

$39.24 \times 10^4 \times 0.07068 + 39.24 \times 10^4 \times 0.07068 \cos 45$

$- F_x = 1000 \times 0.25(-3.537 \cos 45 - 3.537)$.

$47346.319 - F_x = -1509.509$

$F_x = 48855.828\, N$

$$-F_y - P_2A_2\sin 45^\circ = \rho Q(V_2\sin 45 - V_1)$$

$$-F_y - 39.24\times10^4\times0.07068\sin 45^\circ$$

$$= 1000\times0.25(3.537\sin 45 - 3.537)$$

$$\boxed{F_y = -19352.496\,N}$$

-ve sign mean F_y is acting the downward direction.

$$F_r = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{48855.828^2 + 19352.498^2}$$

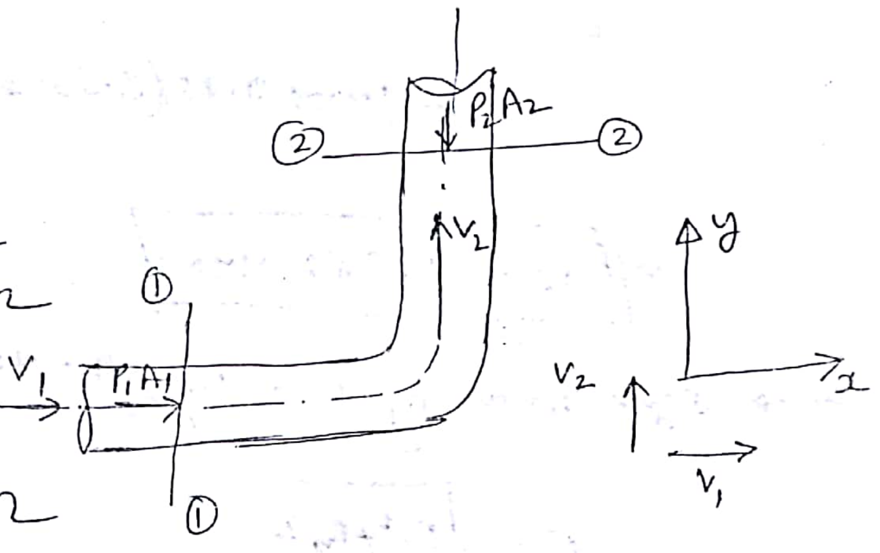
$$\boxed{F_r = 52549.13\,N}$$

$$\tan\theta = \frac{F_y}{F_x} = 21.60^\circ$$

$$= 21^\circ 36'$$

Problem ③ A pipe of 300 mm diameter conveying 0.3 m^3/s of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 24.525 N/cm^2 and 23.544 N/cm^2.

Solution :⇒



$P_1 = 24.525\ N/cm^2$

$P_1 = 24.525 \times 10^4\ N/m^2$

$P_2 = 23.544\ N/cm^2$

$= 23.544 \times 10^4\ N/m^2$

$D = 300mm = 0.3m.$

$A_1 = A_2 = \frac{\pi}{4}(0.3)^2$

$= 0.07068\ m^2.$

$Q = 0.3\ m^3/s.$

$4.244\ m/s = V_1 = V_2$

$Q = A_1 V_1$

$\boxed{V_1 = \frac{Q}{A_1}} = \frac{0.3}{0.07068} = 4.244\ m/s$

$\boxed{\theta = 90}$

$\boxed{\Sigma F = \rho Q (V - u)}$

$P_1 A_1 - F_x = \rho Q (V_2 - V_1)$

$P_1 A_1 - F_x = \rho Q (0 - V_1)$

$24.525 \times 10^4\ 0.07068 - F_x = 1000 \times 0.3(-4.244).$

$+F_x = +18606.27\ N$

$$-F_y - P_2 A_2 = \rho Q_2 (V_2)$$

$$-F_y - 23.544\times10^4 \times 0.07068 = 1000\times0.3 \times (4.244)$$

$$\boxed{F_y = -17914.1\ N}$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$F_R = 25828.43\ N$$

$$\tan\theta = \frac{F_y}{F_x} =$$

$$\theta = 43.91$$

$$= 43^\circ 54'$$

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Flow over Notches & weirs.

Definition:

Notch :- A notch may be defined as an opening provided in the side of a tank or vessel such that the liquid surface in the tank is below the top edge of the opening. A notch may be regarded as an orifice with the water surface below its upper edge.

It is generally made of metallic plate.

It is used for measuring the rate of flow of a liquid through a small channel or a tank.

Weir :- A weir may be defined as any regular obstruction in an open stream over which the flow takes place.

It is made masonary or Concrete.

weirs may be used for measuring the rate of flow of water in rivers or streams.

Note: The main difference b/w a notch and weir is that the notch is of small size but the weir is of a bigger one. Moreover a notch is usually made in a plate, whereas a weir is usually made of masonry or concreate.

Types/Classification of Notches and weirs

Notches

- Rectangular notch.
- Triangular notch.
- Trapezoidal notch
- Stepped Notch.

Types of weirs

(i) According to shape

(i) Rectangular weir

(ii) Cippoletti weir

(ii) According to nature of discharge

(i) ordinary weir

(ii) Submerged or drowned weir

(iii) According to the width of crest.

(i) Narrow - crested weir

(ii) Broad- creasted weir

(iv) According to the nature of crest. ~~Nature to the~~

(i) Sharp- creasted weir

(ii) Ogee weir.

Classification of Mouthpieces.

The mouthpieces may be classified as follows:-

① According to the position of the mouthpiece.

(i) Internal mouthpiece (ii) External mouthpiece.

② According to the Shape of the mouthpiece.

(i) Cylindrical mouthpieces

(ii) Convergent mouthpieces

(iii) Convergent-divergent mouthpieces.

(3) According to the nature of discharge:

(i) Mouthpiece running full (ii) Mouthpiece running free.

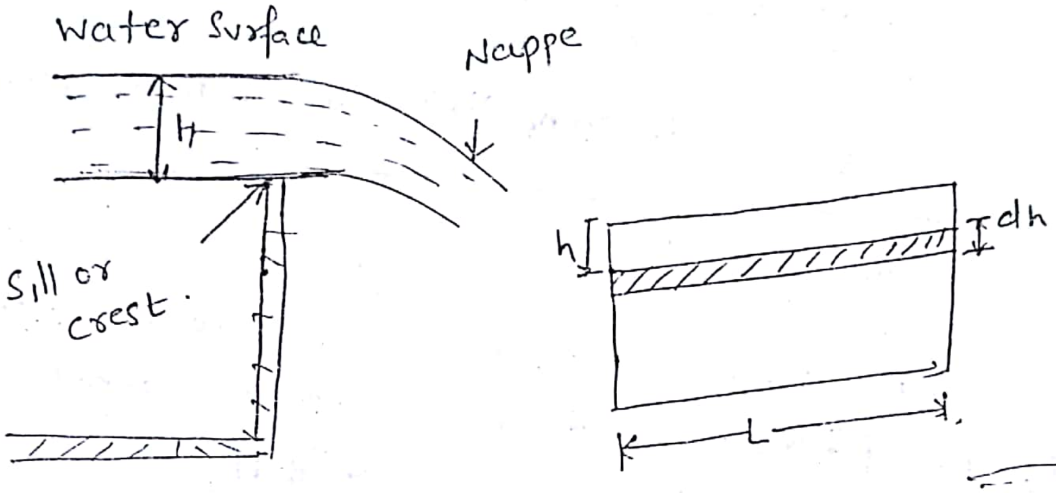
Note: A mouthpiece is said to be running free if the jet of liquid after contraction does not touch the sides of the mouthpiece. But if the jet after contraction expands and fills the whole mouthpiece it is known as running full.

A mouthpieces is an attachment in the form of a small tube or pipe fixed to the orifice (the length of pipe extension is usually 2 to 3 times the orifices diameter). and is used to increase the amount of discharge.

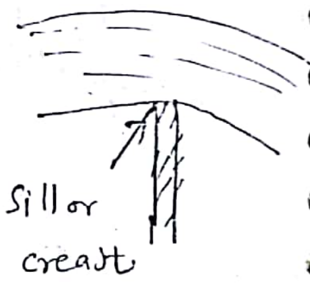
* orifices as well as mouthpieces are used to measure the discharge.

Discharge over a Rectangular Notch or weir.

Consider a rectangular notch or weir provided in a channel carrying water as shown in figure.



(a) Rectangular Notch.



(b) Rectangular weir.

Fig Rectangular Notch and weir.

Let

H = height of water above sill of the notch.

L = length of notch or weir.

C_d = Co-efficient of discharge.

Let us consider a horizontal strip of water thickness dh at a depth h from water level as shown in figure.

Area of Strip $= L \times dh$.

Theoretical velocity of water flowing through strip.

$$= \sqrt{2gh}$$

The discharge through the strip.

$dQ =$ Cd × area of Strip × theoretical velocity.

$$\boxed{dQ = C_d \times L \times dh \times \sqrt{2gh}}$$

The total discharge, over the whole notch, may be found out by integrating the above equation within the limit 0 and H.

$$Q = \int_0^H C_d \times L \times \sqrt{2 \times gh} \times dh$$

$$= C_d \times L \sqrt{2g} \int_0^H (h)^{1/2} \cdot dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g} \left[H \right]^{3/2}$$

$$\boxed{Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \, (H)^{3/2}}$$

Note: The expression for discharge over a rectangular notch or weir is same.

Problem ①

A rectangular notch 2.0 m wide has a constant head of 500 mm. Find the discharge over the notch, if Co-efficient of discharge for the notch is 0.62

Solution:-

Length of the notch $L = 2.0\ m$.

Head over notch $H = 500\ mm = 0.5\ m$.

Co-efficient of discharge $C_d = 0.62$.

Discharge Q

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ (H)^{3/2}.$$

$$= \frac{2}{3} \times 0.62 \times 2 \times \sqrt{2 \times 9.81} \times (0.5)^{3/2}.$$

$$= 1.294\ m^3/s \longrightarrow \text{Ans.}$$

Problem② A rectangular notch has a discharge of 0.24 m^3/s when head of water is 800 mm. Find the length of the notch. Assume $C_d = 0.6$.

Solution:

Discharge $Q = 0.24\ m^3/s$.

Head over notch $H = 800\ mm = 0.8\ m$.

Co-efficient of discharge $C_d = 0.6$.

Length of the notch L

$$Q = \frac{2}{3} C_d \times L \sqrt{2g}\ (H)^{3/2}.$$

~~0.24 × 2~~

$$0.24 = \frac{2}{3} \times 0.6 \times L \times \sqrt{2 \times 9.81} \times (0.8)^{3/2}$$

$$0.24 = 1.267 L$$

$$L = 0.189\ m.$$

$$L = 189\ mm$$ Ans