"WORKING TOWARDS BEING THE BEST"

Name of Faculty: Deepak Kumar Rathaur

Designation: Assistant Professor

Department: Mechanical Engineering

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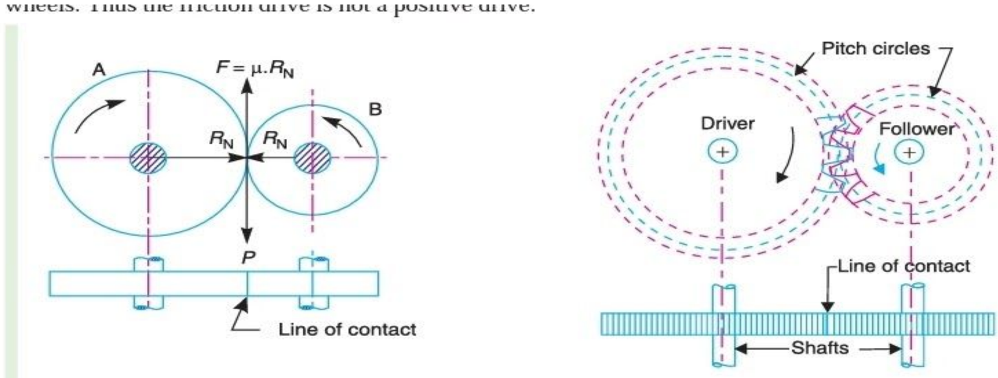
Gear-

We have discussed in the previous chapter, that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of gears or toothed wheels. A gear drive is also provided, when the distance between the driver and the follower is very small.

Friction wheel

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels A and B mounted on shafts, having sufficient rough surfaces and pressing against each other as shown in Fig. 12.1 (a).

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show, that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction as shown in Fig. 12.1 (a).
The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.



(a) Friction wheels. (b) Toothed wheels.

Fig. 12.1

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 12.1 (b), are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as toothed wheel or gear. The usual connection to show the toothed wheels is by their pitch circles.

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Note : Kinematically, the friction wheels running without slip and toothed gearing are identical. But due to the possibility of slipping of wheels, the friction wheels can only be used for transmission of small powers.

Advantages and Disadvantages of Gear Drive --

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

Classification of Toothed Wheels The gears or toothed wheels may be classified as follows :

1. According to the position of axes of the shafts. The axes of the two shafts between which the motion is to be transmitted, may be (a) Parallel,
(b) Intersecting, and
(c) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 12.1. These gears are called spur gears and the arrangement is known as spur gearing. These gears have teeth parallel to the axis of the wheel as shown in Fig. 12.1.
Another name given to the spur gearing is helical gearing, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 12.2 (a) and (b) respectively.

The double helical gears are known as herringbone gears. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact. The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 12.2 (c).
These gears are called bevel gears and the arrangement is known as bevel gearing. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as helical bevel gears.

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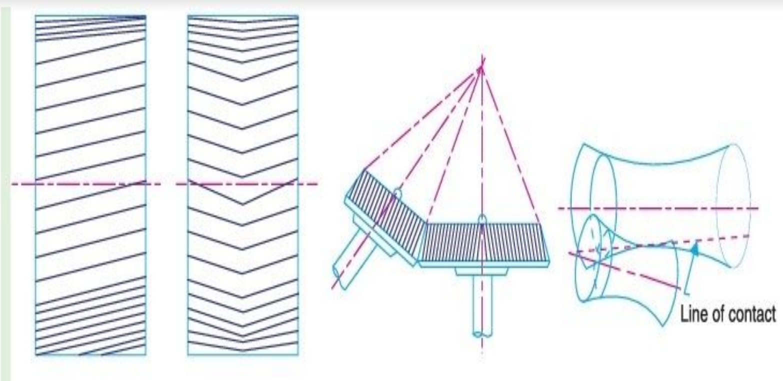
The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears is shown in Fig. 12.2 (d). These gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as hyperboloids.



(a) Single helical gear. (b) Double helical gear. (c) Bevel gear. (d) Spiral gear.

Fig. 12.2

2. According to the peripheral velocity of the gears. The gears, according to the peripheral velocity of the gears may be classified as :

(a) Low velocity,

(b) Medium velocity, and

(c) High velocity.

The gears having velocity less than 3 m/s are termed as low velocity gears and gears having velocity between 3 and 15 m/s are known as medium velocity gears. If the velocity of gears is more than 15 m/s, then these are called high speed gears.

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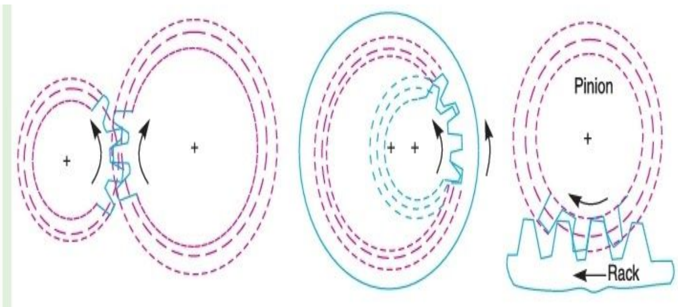
3. According to the type of gearing. The gears, according to the type of gearing may be classified as :
(a) External gearing,
(b) Internal gearing, and
(c) Rack and pinion.
In external gearing, the gears of the two shafts mesh externally with each other as shown in Fig. 12.3 (a). The larger of these two wheels is called spur wheel and the smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always unlike, as shown in Fig. 12.3 (a).

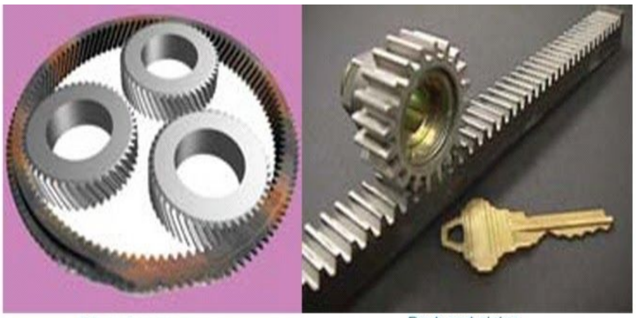


(a) External gearing. (b) Internal gearing.
Fig. 12.3 Fig. 12.4. Rack and pinion.

In internal gearing, the gears of the two shafts mesh internally with each other as shown in Fig. 12.3 (b). The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. In an internal gearing, the motion of the two wheels is always like, as shown in Fig. 12.3 (b).
Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line, as shown in Fig. 12.4. Such type of gear is called rack and pinion. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and vice-versa as shown in Fig. 12.4. 4.

According to position of teeth on the gear surface. The teeth on the gear surface may be (a) straight, (b) inclined, and (c) curved.

We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.



Internal gears Rack and pinion

Terms Used in Gears--

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig. 12.5.

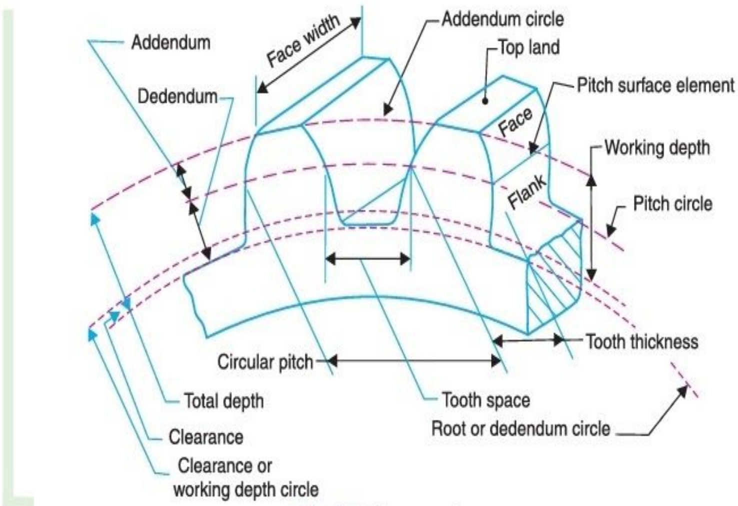


Fig. 12.5. Terms used in gears.

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.
2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usuallyspecified by the pitch circle diameter. It is also known as pitch diameter.
3. Pitch point. It is a common point of contact between two pitch circles.
4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replacedat the pitch circle.
5. Pressure angle or angle of obliquity. It is the angle between the common normal to twogear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ.The standard pressure angles are 1214°and 20°.
6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

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7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric withthe pitch circle.
9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also calledroot circle.
Note : Root circle diameter = Pitch circle diameter × cos φ, where φ is the pressure angle.
10. Circular pitch. It is the distance measured on the circumference of the pitch circle froma point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc.
Mathematically,
Circular pitch $P_c = \pi D/T$
where D=Diameter of the pitch circle, and
T=Number of teeth on the wheel.
A little consideration will show that the two gears will mesh together correctly, if the twowheels have the same circular pitch.

11. Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres.It is denoted by Pd .

12. Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth.It is usually denoted by m. Mathematically,
Module, $m = D/T$
13. Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, ina meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.
14. Total depth. It is the radial distance between the addendum and the dedendum circles ofa gear. It is equal to the sum of the addendum and dedendum.
15. Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.
16. Tooth thickness. It is the width of the tooth measured along the pitch circle.
17. Tooth space . It is the width of space between the two adjacent teeth measured along the pitch circle.
18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.
19. Face of tooth. It is the surface of the gear tooth above the pitch surface.
20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.
21. Top land. It is the surface of the top of the tooth.

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22. Face width. It is the width of the gear tooth measured parallel to its axis.
23. Profile. It is the curve formed by the face and flank of the tooth.
24. Fillet radius. It is the radius that connects the root circle to the profile of the tooth.
25. Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.
26. Length of the path of contact. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.
27. Arc of contact. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e. (a) Arc of approach. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(b) Arc of recess. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as contact ratio i.e. number of pairs of teeth in contact.

Gear Materials

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The nonmetallic materials like wood, raw hide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise. The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important. The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness. The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

Condition for Constant Velocity Ratio of Toothed Wheels–

Law of Gearing Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 12.6. Let the two teethcome in contact at point Q, and the wheels rotate in the directions asshown in the figure.

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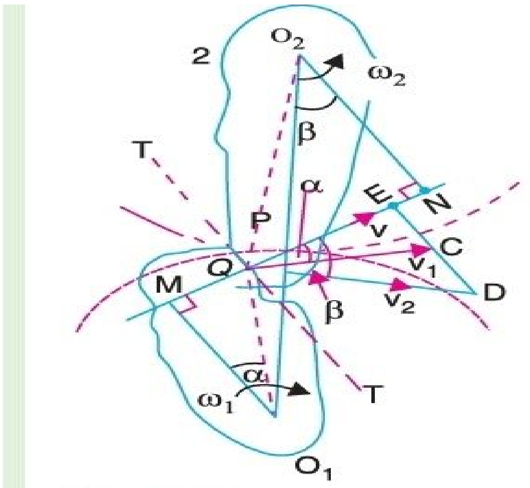

Fig. 12.6. Law of gearing.

Let TT be the common tangent and MN be thecommon normal to the curves at the point of contact Q. From thecentres O1 and O2 , draw O1M and O2N perpendicular to MN. Alittle consideration will show that the point Q moves in the directionQC, when considered as a point on wheel 1, and in the directionQD when considered as a point on wheel 2.

Let v1 and v2 be the velocities of the point Q on the wheels1 and 2 respectively. If the teeth are to remain in contact, then thecomponents of these velocities along the common normal MN mustbe equal.

Fig. 12.6. Law of gearing.

$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

or
$$(\omega_1 \times O_1 Q)\cos\alpha = (\omega_2 \times O_2 Q)\cos\beta$$

$$(\omega_1 \times O_1 Q)\frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q)\frac{O_2 N}{O_2 Q} \quad \text{or} \quad \omega_1 \times O_1 M = \omega_2 \times O_2 N$$

$$\therefore \quad \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}}$$

Also from similar triangles O_1MP and O_2NP,

$$\boxed{\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}}$$

Combining equations (*i*) and (*ii*), we have

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}}$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of thedistances of the point P from the centres O1 and O2, or the common normal to the two

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surfaces at thepoint of contact Q intersects the line of centres at point P which divides the centre distance inverselyas the ratio of angular velocities.Therefore in order to have a constant angular velocity ratio for all positions of the wheels, thepoint P must be the fixed point (called pitch point) for the two wheels. In other words, the commonnormal at the point of contact between a pair of teeth must always pass through the pitch point.

This is the fundamental condition which must be satisfied while designing the profiles for the teeth ofgear wheels. It is also known as law of gearing.

Velocity of Sliding of TeethThe sliding between a pair of teeth in contact at Q occurs along the common tangent T T tothe tooth curves as shown in Fig. 12.6. The velocity of sliding is the velocity of one tooth relative toits mating tooth along the common tangent at the point of contact.

The velocity of point Q, considered as a point on wheel 1, along the common tangent T T isrepresented by EC. From similar triangles QEC and O1MQ,

$$\frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_1 \quad \text{or} \quad EC = \omega_1 . MQ$$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent $T\,T$ is represented by ED. From similar triangles QCD and $O_2\,NQ$,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2 . QN$$

Let v_S = Velocity of sliding at Q.

$$\therefore \quad v_S = ED - EC = \omega_2 .\, QN - \omega_1 . MQ$$

$$= \omega_2\,(QP + PN) - \omega_1\,(MP - QP)$$

$$= (\omega_1 + \omega_2)\,QP + \omega_2 .\,PN - \omega_1 . MP \qquad \ldots(i)$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$ or $\omega_1 .\, MP = \omega_2 . PN$, therefore equation (*i*) becomes

$$v_S = (\omega_1 + \omega_2)\,QP \qquad \ldots(ii)$$

Notes :1.We see from equation (ii), that the velocity of sliding is proportional to the distance of the pointof contact from the pitch point.

2.Since the angular velocity of wheel 2 relative to wheel 1 is ($\omega 1 + \omega 2$) and P is the instantaneouscentre for this relative motion, therefore the value of vs may directly be written as vs ($\omega 1 + \omega 2$) QP, without theabove analysis.

Forms of Teeth-

We have discussed that conjugate teeth are not in common use.Therefore, in actual practice following are the twotypes of teeth commonly used :

1. Cycloidal teeth ; and
2. Involute teeth.

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1)Cycloidal Teeth

A cycloid is the curve traced by a point on the circumference of a circle which rolls withoutslipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle,the curve traced by a point on the circumference of a circle is known as epi-cycloid. On the otherhand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a pointon the circumference of a circle is called hypo-cycloid.

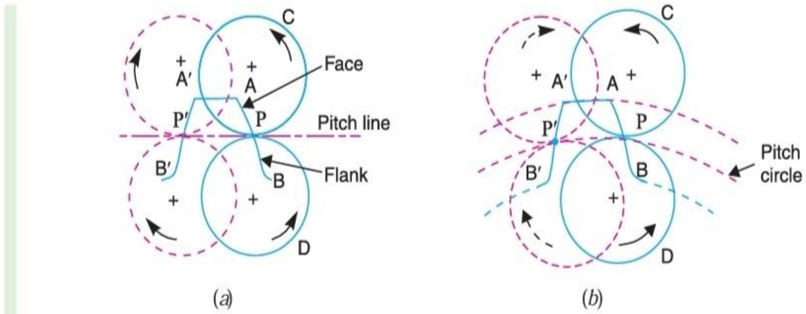

Fig. 12.7. Construction of cycloidal teeth of a gear.

2) Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 12.9. In connection with toothed wheels, the circle is known as base circle.

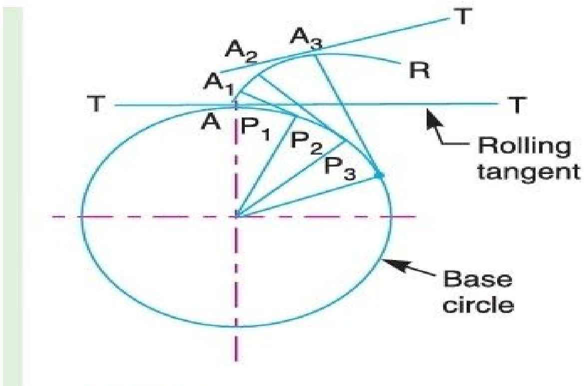

Fig. 12.9. Construction of involute.

Standard Proportions of Gear Systems

The following table shows the standard proportions in module (m) for the four gear systemsas discussed in the previous article.

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Table 12.1. Standard proportions of gear systems.

S. No.	Particulars	$14\frac{1}{2}°$ composite or full depth involute system	20° full depth involute system	20° stub involute system
1.	Addenddm	1 *m*	1 *m*	0.8 *m*
2.	Dedendum	1.25 *m*	1.25 *m*	1 *m*
3.	Working depth	2 *m*	2 *m*	1.60 *m*
4.	Minimum total depth	2.25 *m*	2.25 *m*	1.80 *m*
5.	Tooth thickness	1.5708 *m*	1.5708 *m*	1.5708 *m*
6.	Minimum clearance	0.25 *m*	0.25 *m*	0.2 *m*
7.	Fillet radius at root	0.4 *m*	0.4 *m*	0.4 *m*

Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig. 12.11. When the pinion rotates inclockwise direction, the contact between a pair of involute teeth begins at K (on the flank near thebase circle of pinion or the outer end of the tooth face on the wheel) and ends at L (outer end of thetooth face on the pinion or on the flank near the base circle of wheel). MN is the common normal atthe point of contacts and the common tangent to the base circles. The point K is the intersection of theaddendum circle of wheel and the common tangent.

The point L is the intersection of the addendum circle of pinion and common tangent.

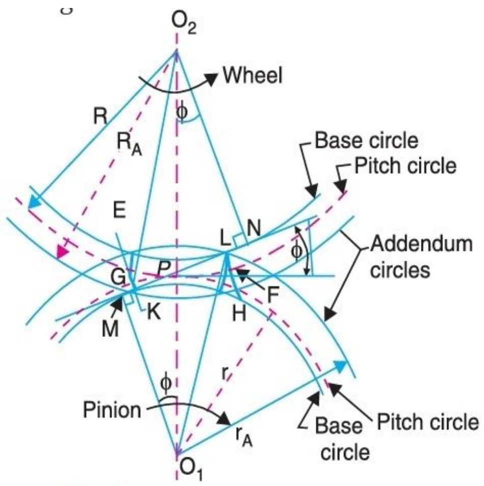

Fig. 12.11. Length of path of contact.

We have discussed that the length of path of contact is the length of common normal cutoff by the addendum circles of the wheel and the pinion.

Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as path of approach and the part of the path of contact PL is known as path of recess.

Let rA = O1L = Radius of addendum circle of pinion,

RA = O2K = Radius of addendum circle of wheel,

r = O1P = Radius of pitch circle of pinion, and Bevel gear

R = O2P = Radius of pitch circle of wheel.

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From Fig. 12.11, we find that radius of the base circle of pinion,

$$O_1M = O_1P\cos\phi = r\cos\phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P\cos\phi = R\cos\phi$$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

and $$PN = O_2P\sin\phi = R\sin\phi$$

$\therefore$ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angled triangle O_1ML,

and $$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

$\therefore$ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

Length of Arc of Contact

We have already defined that the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 12.11, the arc of contact is EPF or GPH. Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as arc of approach and the arc PH is called arc of recess. The angles subtended by these arcs at O1 are called angle of approach and angle of recess respectively.

We know that the length of the arc of approach (arc *GP*)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc *PH*)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Since the length of the arc of contact *GPH* is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

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Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

Mathematically,

Contact ratio or number of pairs of teeth in contact =Length of the arc of contact / Pc

where Pc= circular pitch = π m

and m = module

Notes : 1. The contact ratio, usually, is not a whole number. For example, if the contact ratio is 1.6, it does not mean that there are 1.6 pairs of teeth in contact. It means that there are alternately one pair and two pairs of teeth in contact and on a time basis the average is 1.6.

2. The theoretical minimum value for the contact ratio is one, that is there must always be at least one pair of teeth in contact for continuous action.

3. Larger the contact ratio, more quietly the gears will operate.

GEAR TRAIN:-

When two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels. The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

1) Simple Gear Train

When there is only one gear on each shaft, as shown in Fig. 13.1, it is known as simple gear train. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. 13.1 (a). Since the gear 1 drives the gear 2, therefore gear 1 is called the

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driver and the gear 2 is called the driven or follower. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

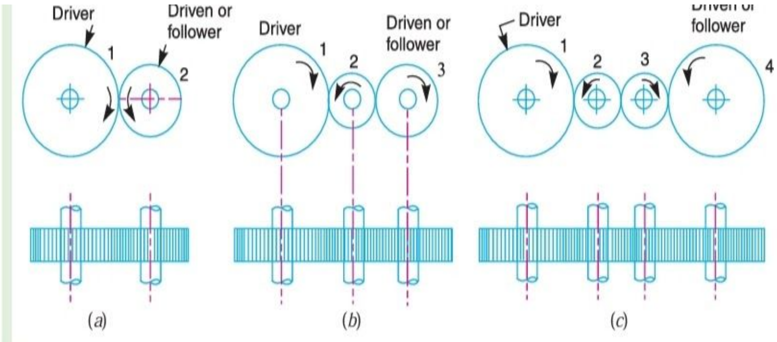

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Fig. 13.1. Simple gear train.

Let N1 = Speed of gear 1(or driver) in r.p.m.,

N2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T1 = Number of teeth on gear 1, and

T2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

Speed ratio = N1/N2=T2/T1

and Train value= N2/N1=T1/T2

Now consider a simple train of gears with one intermediate gear as shown in Fig. 13.1 (b).

Let N1 = Speed of driver in r.p.m.,

N2 = Speed of intermediate gear in r.p.m.,

N3 = Speed of driven or follower in r.p.m.,

T1 = Number of teeth on driver,

T2 = Number of teeth on intermediate gear, and T3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{...}(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

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$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \text{...}(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations (*i*) and (*ii*).

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

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2)Compound Gear Train

Gear trains inside a mechanical watch When there are more than one gear on a shaft, as shown in Fig. 13.2, it is called a compound train of gear.

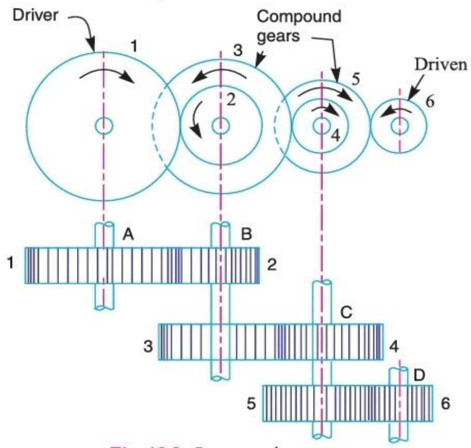

Fig. 13.2. Compound gear train.

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In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$

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Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (i), (ii) and (iii),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad ^{*}\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

3)Reverted Gear Train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train as shown in Fig. 13.4.

We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound

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gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is like.

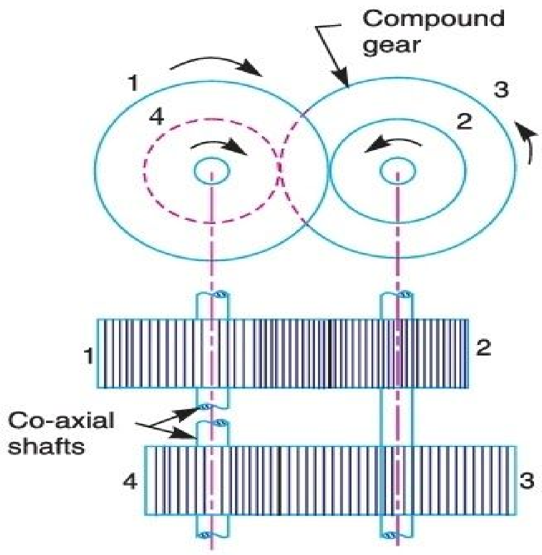

Fig. 13.4. Reverted gear train.

1,

Let, T1 = Number of teeth on gear 1,
r1 = Pitch circle radius of gear 1, and
N1 = Speed of gear 1 in r.p.m.
Similarly,
T2, T3, T4 = Number of teeth on respective gears,
r2, r3, r4 = Pitch circle radii of respective gears, and N2, N3, N4 = Speed of respective gears in r.p.m.
Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4is same, thereforer1 + r2 = r3 + r4...(i)
Also, the circular pitch or module of all the gears is assumed to be same, therefore number ofteeth on each gear is directly proportional to its circumference or radius.

4) Epicyclic Gear Train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over whichthe gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown inFig. 13.6, where a gear A and the arm C have a common axis at O1 about which they can rotate. Thegear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice- versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are

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known as epicyclic gear trains (epi. means upon and cyclic means around). The epicyclic gear trains may be simple or compound. The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

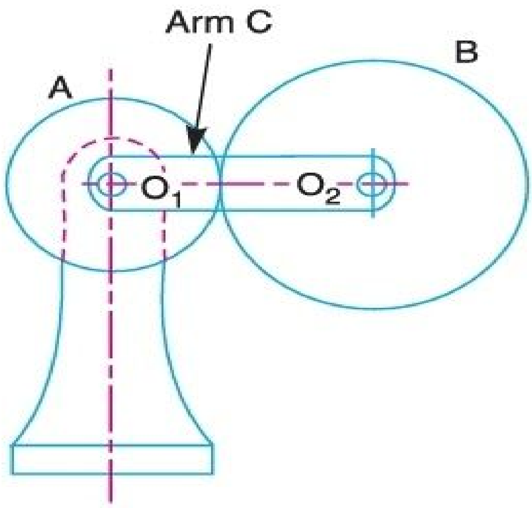

Fig. 13.6. Epicyclic gear train.

Velocity Ratioz of Epicyclic Gear Train

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular method, and
2. Algebraic method.

These methods are discussed, in detail, as follows :

1. Tabular method.

Consider an epicyclic gear train as shown in Fig. 13.6.

Let TA = Number of teeth on gear A, and

TB = Number of teeth on gear B.

First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other. When the gear A makes one revolution anticlockwise, the gear B will make TA / TB revolutions, clockwise. Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes + 1 revolution, then the gear B will make (– TA / TB) revolutions. This statement of relative motion is entered in the first row of the table (see Table 13.1).

`

Table 13.1. Table of motions

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	$+1$	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Secondly, if the gear A makes + x revolutions, then the gear B will make – x × TA / TB revolutions. This Inside view of a car engine. statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by x. Note : This picture is given as additional information and is not a direct example of the current chapter. Thirdly, each element of an epicyclic train is given + y revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

2. Algebraic method.

In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train viz. some element is fixed and the other has specified motion. These two conditions are sufficient to solve all the equations ; and hence to determine the motion of any element in the epicyclic gear train. Let the arm C be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear A relative to the arm C = NA – NC

Let the arm C be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear A relative to the arm C

$$= N_A - N_C$$

and speed of the gear B relative to the arm C,

$$= N_B - N_C$$

Since the gears A and B are meshing directly, therefore they will revolve in ***opposite*** directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm C is fixed, therefore its speed, $N_C = 0$.

$$\therefore \quad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.

Example 13.7. *An epicyclic train of gears is arranged as shown in Fig.13.11. How many revolutions does the arm, to which the pinions B and C are attached, make :*

1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and

2. when A makes one revolution clockwise and D is stationary ?

The number of teeth on the gears A and D are 40 and 90 respectively.

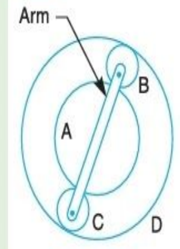

Fig. 13.11

Solution. Given : $T_A = 40$; $T_D = 90$

First of all, let us find the number of teeth on gears *B* and *C* (*i.e.* T_B and T_C). Let d_A, d_B, d_C and d_D be the pitch circle diameters of gears *A*, *B*, *C* and *D* respectively. Therefore from the geometry of the figure,

$$d_A + d_B + d_C = d_D \quad \text{or} \quad d_A + 2\,d_B = d_D \qquad ...(\because d_B = d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \quad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(\because T_B = T_C)$$

The table of motions is given below :

Table 13.6. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B} \times \frac{T_B}{T_D} = +\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions	0	– *x*	$+x \times \frac{T_A}{T_B}$	$+x \times \frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	– *y*	– *y*	– *y*	– *y*
4.	Total motion	– *y*	– *x* – *y*	$x \times \frac{T_A}{T_B} - y$	$x \times \frac{T_A}{T_D} - y$

1. *Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise*

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1 \quad \text{or} \quad x + y = 1 \qquad ...(i)$$

Also, the gear *D* makes half revolution anticlockwise, therefore

$$x \times \frac{T_A}{T_D} - y = \frac{1}{2} \quad \text{or} \quad x \times \frac{40}{90} - y = \frac{1}{2}$$

$$\therefore \quad 40\,x - 90\,y = 45 \quad \text{or} \quad x - 2.25\,y = 1.125 \qquad ...(ii)$$

From equations (*i*) and (*ii*), $x = 1.04$ and $y = -\,0.04$

$\therefore$ Speed of arm $= -\,y = -\,(-\,0.04) = +\,0.04$

$= 0.04$ revolution anticlockwise **Ans.**

2. *Speed of arm when A makes 1 revolution clockwise and D is stationary*

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1 \quad \text{or} \quad x + y = 1 \qquad ...(iii)$$

Also the gear *D* is stationary, therefore

$$x \times \frac{T_A}{T_D} - y = 0 \quad \text{or} \quad x \times \frac{40}{90} - y = 0$$

$$\therefore \quad 40\,x - 90\,y = 0 \quad \text{or} \quad x - 2.25\,y = 0 \qquad ...(iv)$$

From equations (*iii*) and (*iv*),

$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$\therefore$ Speed of arm $= -\,y = -\,0.308 = 0.308$ revolution clockwise **Ans.**

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BALANCING-

Introduction-- The balancing of rotating bodies is important to avoid vibration. In heavy industrial machines such as gas turbines and electric generators, vibration can cause catastrophic failure, as well as noise and discomfort. In the case of a narrow wheel, balancing simply involves moving the center of gravity to the centre of rotation. For a system to be in complete balance both force and couple polygons should be closed.in order to prevent the effect of centrifugal force. Balancing is important to design the machine part's so wisely, that the unbalance is reduced up to the minimum possible level or eliminated completely.

Types of balancing-

1)Static balancing

2) Dynamic balancing

1) Static balancing- Static balance occurs when the centre of gravity of an object is on the axis of rotation. The object can therefore remain stationary, with the axis horizontal, without the application of any braking force. It has no tendency to rotate due to the force of gravity. This is seen in bike wheels where the reflective plate is placed opposite the valve to distribute the centre of mass to the centre of the wheel. Other examples are grindstones, discs or car wheels.
2) Dynamic balancing-A rotating system of mass is in dynamic balance when the rotation does not produce any resultant centrifugal force or couple. The system rotates without requiring the application of any external force or couple, other than that required to support its weight. If a system is initially unbalanced, to avoid the stress upon the bearings caused by the centrifugal couple, counterbalancing weights must be added. This is seen when a bicycle wheel gets buckled. The wheel will not rotate itself when stationary due to gravity as it is still statically balanced, but will not rotate smoothly as the centre of mass is to the side of the centre bearing. The spokes on a bike wheel need to be tuned in order to stop this and keep the wheel operating as efficiently as possible.

Difference between static and dynamic balancing-

Static balance refers to the ability of a stationary on object to its balance. This happens when the objects centre of gravity is on the axis of rotation. Whereas dynamic balance is the ability of an object to balance whilst in motion or when switching between positions.

For any form of balance to happen, the centre of gravity must be aligned over the objects support base. The centre of gravity refers to the part which is the centre of an object weight. Balancing plays a very important part in machines. Balancing in machines helps to rotating bodies to avoid vibrations; vibration in machines can lead to failure. Common failure occurs in generators and heavy machinery, so undertaking in balancing can help to avoid machines from breaking down.

Balancing can also involves shifting the centre of gravity towards the centre of rotation. Dynamic balancing is when the rotating system doesn't yield any other force or couple. Other than the force that is needed the system will rotate without the need for any additional external force or pressure to be applied.

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Static balancing definition refers to the ability of a stationary object to its balance. The occurs when a parts centre of gravity is on the axis of rotation. However, the dynamic balance definition is the ability of an object to balance whilst in motion or when switching between positions.

Undertaking in balancing whether it be static or dynamic can help to extend the service life, quality and accuracy of your machinery. Unbalanced parts can lead to your machine breaking down or worst of all catastrophic failure.

VIBRATION:-

Introduction-Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. The word comes from Latin vibrationem ("shaking, brandishing"). The oscillations may be periodic, such as the motion of a pendulum—or random, such as the movement of a tire on a gravel road.

Vibration can be desirable: for example, the motion of a tuning fork, the reed in a woodwind instrument or harmonica, a mobile phone, or the cone of a loudspeaker.

In many cases, however, vibration is undesirable, wasting energy and creating unwanted sound. For example, the vibrational motions of engines, electric motors, or any mechanical device in operation are typically unwanted. Such vibrations could be caused by imbalances in the rotating parts, uneven friction, or the meshing of gear teeth. Careful designs usually minimize unwanted vibrations.

The studies of sound and vibration are closely related. Sound, or pressure waves, are generated by vibrating structures (e.g. vocal cords); these pressure waves can also induce the vibration of structures (e.g. ear drum). Hence, attempts to reduce noise are often related to issues of vibration.

Types of Vibration--
Free vibration- Free vibration occurs when a mechanical system is set in motion with an initial input and allowed to vibrate freely. Examples of this type of vibration are pulling a child back on a swing and letting it go, or hitting a tuning fork and letting it ring. The mechanical system vibrates at one or more of its natural frequencies and damps down to motionlessness.

Forced vibration-- Forced vibration is when a time-varying disturbance (load, displacement or velocity) is applied to a mechanical system. The disturbance can be a periodic and steady-state input, a transient input, or a random input. The periodic input can be a harmonic or a non-harmonic disturbance. Examples of these types of vibration include a washing machine shaking due to an imbalance, transportation vibration caused by an engine or uneven road, or the vibration of a building during an earthquake. For linear systems, the frequency of the steady-state vibration response resulting from the application of a periodic, harmonic input is equal to the frequency of the applied force or motion, with the response magnitude being dependent on the actual mechanical system.

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Damped vibration:- When the energy of a vibrating system is gradually dissipated by friction and other resistances, the vibrations are said to be damped. The vibrations gradually reduce or change in frequency or intensity or cease and the system rests in its equilibrium position. An example of this type of vibration is the vehicular suspension dampened by the shock absorber.

Vibration Analysis-

Vibration Analysis (VA), applied in an industrial or maintenance environment aims to reduce maintenance costs and equipment downtime by detecting equipment faults. VA is a key component of a condition monitoring (CM) program, and is often referred to as predictive maintenance (PdM).Most commonly VA is used to detect faults in rotating equipment (Fans, Motors, Pumps, and Gearboxes etc.) such as Unbalance, Misalignment, rolling element bearing faults and resonance conditions.

VA can use the units of Displacement, Velocity and Acceleration displayed as a time waveform (TWF), but most commonly the spectrum is used, derived from a fast Fourier transform of the TWF. The vibration spectrum provides important frequency information that can pinpoint the faulty component.

The fundamentals of vibration analysis can be understood by studying the simple Mass-spring-damper model. Indeed, even a complex structure such as an automobile body can be modeled as a "summation" of simple mass–spring–damper models. The mass–spring–damper model is an example of a simple harmonic oscillator. The mathematics used to describe its behavior is identical to other simple harmonic oscillators such as the RLC circuit.

Single Degree of Freedom(SDOF)--

Single Degree of Freedom (SDOF) system
1. Equation of motion (EOM)
Mathematical expression defining the dynamic displacements of a structural system. Solution of the expression gives a complete description of the response of the structure as a function of time

Derivation of EOM
1. Dynamic Equilibrium (Using D'Alembert's principle)
2. Principle of Virtual Work
3. Hamilton's principle (Using Lagrange's equation)
Dynamic Equilibrium
D'Alembert's principle states that a mass develops an inertial force proportional to its acceleration and opposing its motion. (See Figure 3)
$m\ddot{u} + ku = F(t)$ Equation of Motion (1)
for $F(t) = 0$, the response is termed as free vibration and occurs due to initial excitation.

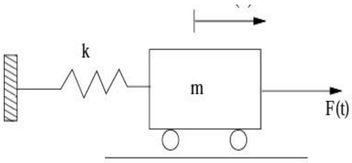

Figure 1: Undamped SDOF system

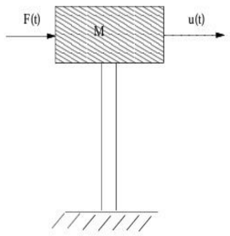

Figure 2: Example of overhead water tank that can be modeled as SDOF system

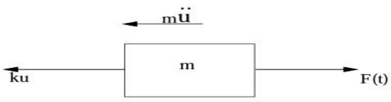

Figure 3: Dynamic force equilibrium

<u>**Free Vibration**</u>

$$
\begin{aligned}
& m\ddot{u} + ku = 0 \quad \text{linear,homogeneous second order differential equation} \\
& \Rightarrow \ddot{u} + \frac{k}{m}u = 0 \\
& \Rightarrow \ddot{u} + \omega_n^2 u = 0 \quad \omega_n^2 = \frac{k}{m},\ \omega_n = \sqrt{\frac{k}{m}} \quad \omega_n = \text{natural frequency}
\end{aligned} \tag{2}
$$

Solution of Equation 2 will be,

$$
\begin{aligned}
u(t) &= C_1 e^{\imath\omega_n t} + C_2 e^{-\imath\omega_n t} \\
&= C_1(\cos\omega_n t + \imath\sin\omega_n t) + C_2(\cos\omega_n t - \imath\sin\omega_n t) \\
&= (C_1 + C_2)\cos\omega_n t + \imath(C_1 - C_2)\sin\omega_n t
\end{aligned} \tag{3}
$$

Applying the initial conditions,

$$
\begin{aligned}
u(t)|_{@t=0} &= u_0 = C_1 + C_2 \\
\dot{u}(t)|_{@t=0} &= \dot{u}_0 = \imath\omega_n(C_1 - C_2)
\end{aligned} \tag{4}
$$

Substituting Equation 4 into Equation 3, we get,

$$
u(t) = u_0\cos\omega_n t + \frac{\dot{u}_0}{\omega_n}\sin\omega_n t \tag{5}
$$

Again, substituting,

$$
\begin{aligned}
u_0 &= A\cos\phi \\
\frac{\dot{u}_0}{\omega_n} &= A\sin\phi
\end{aligned} \tag{6}
$$

into Equation 5, we get,

$$
\begin{aligned}
u(t) &= A\cos\phi\cos\omega_n t + A\sin\phi\sin\omega_n t \\
&= A\cos(\omega_n t - \phi)
\end{aligned} \tag{7}
$$

where, A is the *amplitude* and ϕ is the *phase angle*

$$A = \sqrt{u_0^2 + \left(\frac{\dot{u}_0}{\omega_n}\right)^2} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{\dot{u}_0/\omega_n}{u_0}\right) \tag{8}$$

Free vibration of damped SDOF system

Modeling of damping is perhaps one of the most difficult task in structural dynamics. It is still a topic of research in advanced structural dynamics and is derived mostly experimentally.

Viscous Damping

The most common form of damping is viscous damping.

Equation of Motion

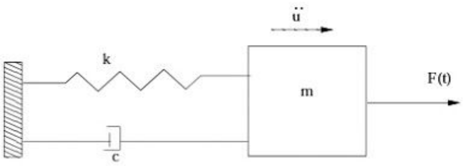

Figure 4: SDOF with viscous damping

$$\begin{aligned} & m\ddot{u} + c\dot{u} + ku = 0 \\ & \Rightarrow \ddot{u} + \frac{c}{m}\dot{u} + \frac{k}{m}u = 0 \\ & \Rightarrow \ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = 0 \end{aligned} \tag{9}$$

where, $\xi = \frac{c}{2m\omega_n}$ is the *viscous damping factor*. Assuming a solution $u(t) = Ce^{st}$ and substituting in Equation 9, we get,

$$\begin{aligned} & s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \\ & \Rightarrow s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} \\ & = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_n \end{aligned} \tag{10}$$

Depending on the value of ξ, the nature of s and correspondingly $u(t)$ will be determined,

$$\begin{aligned} u(t) &= C_1 e^{(-\xi + \sqrt{\xi^2 - 1})\omega_n t} + C_2 e^{(-\xi - \sqrt{\xi^2 - 1})\omega_n t} \\ &= \left[C_1 e^{\sqrt{(\xi^2-1)}\omega_n t} + C_2 e^{-\sqrt{(\xi^2-1)}\omega_n t}\right] e^{-\xi\omega_n t} \end{aligned} \tag{11}$$

<u>**Case I**</u> *Under-damped system,* $0 < \xi < 1$
For $\xi < 1$, $s_1,\ s_2$ are complex numbers and given as,

$$s_1\ s_2 = \left(-\xi \pm \imath\sqrt{|\xi^2 - 1|}\right)\omega_n \tag{12}$$

Therefore,

$$u(t) = \left(C_1 e^{\imath\sqrt{|\xi^2-1|}\omega_n t} + C_2 e^{-\imath\sqrt{|\xi^2-1|}\omega_n t}\right)e^{-\xi\omega_n t} \tag{13}$$

Considering $\sqrt{|\xi^2 - 1|}\omega_n = \omega_d$, Equation 13 can be written as,

$$\begin{aligned} u(t) &= \left(C_1 e^{\imath\omega_d t} + C_2 e^{-\imath\omega_d t}\right)e^{-\xi\omega_n t} \\ &[(C_1 + C_2)\cos\omega_d t + \imath(C_1 - C_2)\sin\omega_d t]\, e^{-\xi\omega_n t} \end{aligned} \tag{14}$$

where, ω_d is referred as *damped natural frequency.* Substituting $(C_1 + C_2) = A\cos\phi$ and $\imath(C_1 - C_2) = A\sin\phi$ into Equation 14, we get,

$$u(t) = A\cos(\omega_d t - \phi)e^{-\xi\omega_n t} \tag{15}$$

Applying initial conditions as, $u(t)|_{@t=0} = u_0$ and $\dot{u}(t)|_{@t=0} = \dot{u}_0$, we get,

$$C_1 + C_2 = u_0 \quad \text{and} \quad \imath(C_1 - C_2) = \left[\frac{\dot{u}_0}{\omega_d} + \frac{u_0\xi}{\sqrt{1-\xi^2}}\right]$$

Thus for these initial conditions, the response can be written as,

$$u(t) = \left(u_0\cos\omega_d t + \left[\frac{\dot{u}_0}{\omega_d} + \frac{u_0\xi}{\sqrt{1-\xi^2}}\right]\sin\omega_d t\right)e^{-\xi\omega_n t} \tag{16}$$

<u>**Case II**</u> *Critically-damped system,* $\xi = 1$
Critical damping is the minimum damping required to stop the oscillations.

$$s_1,\ s_2 = -\omega_n$$

The solution is of the form,

$$u(t) = (C_1 + C_2 t)e^{-\omega_n t} \tag{17}$$

Even here, C_1 and C_2 can be obtained from the initial conditions given.
<u>**Case III**</u> *Over-damped system,* $\xi > 1$
There is no oscillatory motion in an over-damped system.

$$u(t) = (C_1 e^{\omega_d t} + C_2 e^{-\omega_d t})e^{-\xi\omega_n t} \tag{18}$$

For a over-damped system, higher the values of ξ, the slower the rate of the decay (See Figure 5).