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Subject: MCD (ME-602)

Unit: 3

Topic: Spring

Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

2. To apply forces, as in brakes, clutches and spring-loaded valves.

3. To control motion by maintaining contact between two elements as in cams and followers.

4. To measure forces, as in spring balances and engine indicators.

5. To store energy, as in watches, toys, etc.

Types of spring

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

1. ***Helical springs*.** The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are

(*a*) Compression helical spring. (*b*) Tension helical spring.

compression helical spring as shown in Fig (*a*) and ***tension helical spring*** as shown in Fig (*b*).

Helical springs.

The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

In ***open coiled helical springs*****,** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(a) These are easy to manufacture.

(b) These are available in wide range.

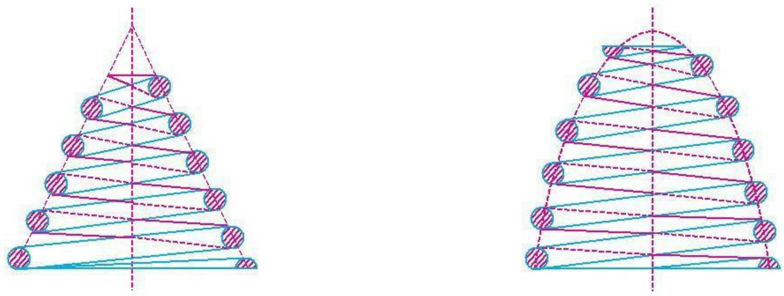
(c) These are reliable.

(d) These have constant spring rate.

(e) Their performance can be predicted more accurately.

(f) Their characteristics can be varied by changing dimensions.

2. ***Conical and volute springs*.** The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig (*a*), is wound with a uniform pitch whereas the

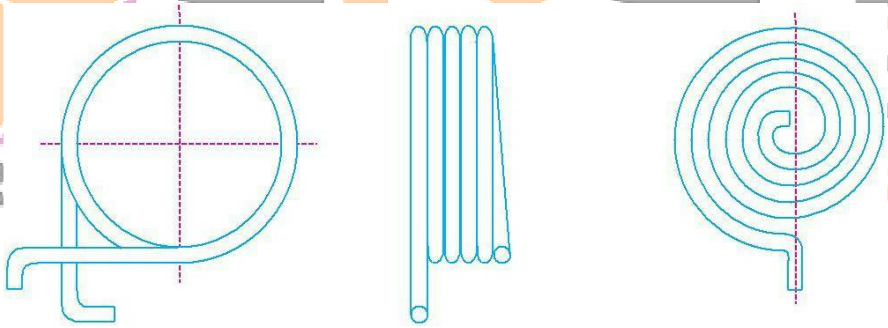


(*a*) Conical spring. (*b*) Volute spring.

volute springs, as shown in Fig. (*b*), are wound in the form of paraboloid with constant pitch and lead angles. The springs may be made either partially or completely telescoping. In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate. This characteristic is sometimes utilised in vibration problems where springs are used to support a body that has a varying mass.

The major stresses produced in conical and volute springs are also shear stresses due to twisting.

3. ***Torsion springs*.** These springs may be of ***helical*** or ***spiral*** type as shown in Fig. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.



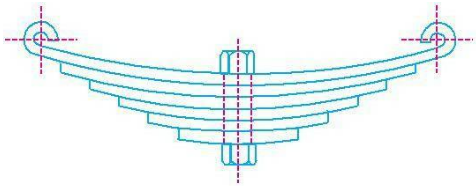
The major stresses produced in torsion springs are tensile and compressive due to bending.

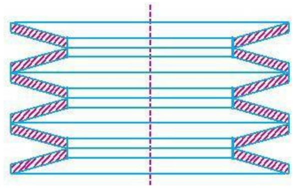
(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

Torsion springs.

4. ***Laminated or leaf springs*.** The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. These are mostly used in automobiles.

The major stresses produced in leaf springs are tensile and compressive stresses.





Laminated or leaf springs Disc or bellevile springs

5. ***Disc or bellevile springs*.** These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

6. ***Special purpose springs.*** These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

Uses of springs:

(a) To apply forces and to control motions as in brakes and clutches.

(b) To measure forces as in spring balance.

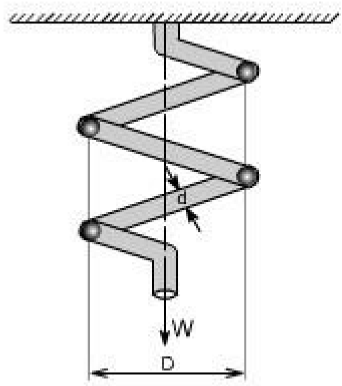
(c) To store energy as in clock springs.

(d) To reduce the effect of shock or impact loading as in carriage springs.

(e) To change the vibrating characteristics of a member as inflexible mounting of motors.

<u>Derivation of the Formula :</u>

In order to derive a necessary formula which governs the behaviour of springs, consider a closed coiled spring subjected to an axial load W.



Let

W = axial load

D = mean coil diameter

d = diameter of spring wire n =

number of active coils

C = spring index = D / d for circular wires l =

length of spring wire

G = modulus of rigidity x =

deflection of spring q =

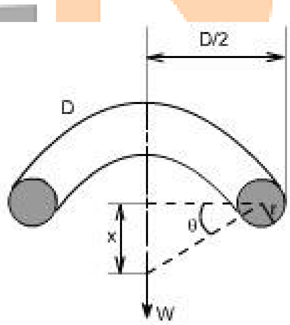
Angle of twist

When the spring is being subjected to an axial load to the wire of the spring gets be twisted like a shaft.

If q is the total angle of twist along the wire and x is the deflection of spring under the action of load W along the axis of the coil, so that

x = D*q / 2.

Again l = p D n [consider, one half turn of a close coiled helical spring]



Assumptions: (1) The Bending & shear effects may be neglected.

(2) For the purpose of derivation of formula, the helix angle is considered to be so small that it may be neglected.

Any one coil of a spring will be assumed to lie in a plane which is nearly ^r to the axis of the spring. This requires that adjoining coils be close together. With this limitation, a section taken perpendicular to the axis the spring rod becomes nearly vertical. Hence to maintain equilibrium of a segment of the spring, only a shearing force V = F and Torque T = F. r are required at any X – section. In the analysis of springs it is customary to assume that the shearing stresses caused by the direct shear force is uniformly distributed and is negligible

So applying the torsion formula.
Using the torsion formula i.e

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$$

$$\text{and substitituting } J = \frac{\pi d^4}{32}; T = w.\frac{d}{2}$$

$$\theta = \frac{2.x}{D}; l = \pi D.x$$

$$\frac{w.d/2}{\frac{\pi d^4}{32}} = \frac{G.2x/D}{\pi D.n}$$

Thus,

$$x = \frac{8w.D^3.n}{G.d^4}$$

Spring stiffness: The stiffness is defined as the load per unit deflection therefore

$$k = \frac{W}{x} = \frac{W}{\frac{8wD^3.n}{G.d^4}}$$

Therefore

$$k = \frac{G.d^4}{8.D^3.n}$$

WAHL'S FACTOR:

In order to take into account the effect of direct shear and change in coil curvature a stress factor is defined, which is known as Wahl's factor

$$K = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

K = Wahl"s factor and is defined as Where C = spring index = D/d

If we take into account the Wahl's factor than the formula for the shear stress

$$\tau_{max} = \frac{16.T.k}{\pi d^3}$$

Strain Energy: The strain energy is defined as the energy which is stored within a material when the work has been done on the material.

In the case of a spring the strain energy would be due to bending and the strain energy due to bending is given by the expansion

$$U = \frac{T^2L}{2EI}$$

$$L = \pi D n$$

$$I = \frac{\pi d^4}{64}$$

so after substitution we get

$$U = \frac{32T^2Dn}{E.d^4}$$

Example: A close coiled helical spring is to carry a load of 5000N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm^2 .if the number of active turns or active coils is 8.Estimate the following:

(i) Wire diameter

(ii) Mean coil diameter

(iii) Weight of the spring.

Assume G = 83,000 N/mm^2 ; r = 7700 kg/m^3

Solution:

(i) For wire diameter if W is the axial load, then

$$\frac{\frac{w.d/2}{\frac{\pi d^4}{32}}} = \frac{\tau_{max}m}{d/2}$$

$$D = \frac{400}{d/2} \cdot \frac{\pi d^4}{32} \cdot \frac{2}{W}$$

$$D = \frac{400.\pi d^3.2}{5000.16}$$

$$D = 0.0314\, d^3$$

(ii) Further, deflection is given as

$$x = \frac{8wD^3.n}{G.d^4}$$

on substituting the relevant parameters we get

$$50 = \frac{8.5000.(0.0314\, d^3)^3.8}{83{,}000.d^4}$$

$$d = 13.32\text{mm}$$

Therefore,

(iii) D = .0314 x (13.317)^{3}mm

=74.15mm D =
74.15 mm

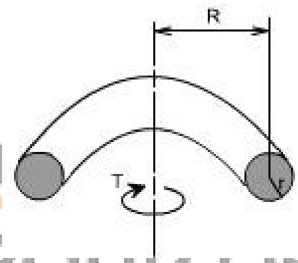
(iv) **Weight**

$$\text{mass or weight} = \text{volume.density}$$
$$= \text{area.length of the spring.density of spring material}$$
$$= \frac{\pi d^2}{4}.\pi D n.\rho$$

On substituting the relevant parameters we get

$$\text{Weight} = 1.996\ \text{kg}$$
$$\simeq 2.0\ \text{kg}$$

(v) **Close – coiled helical spring subjected to axial torque T or axial couple.**



In this case the material of the spring is subjected to pure bending which tends to reduce Radius R of the coils. In this case the bending moment is constant through out the spring and is equal to the applied axial Torque T. The stresses i.e. maximum bending stress may thus be determined from the bending theory.

$$\sigma_{max} = \frac{M.y}{I}$$
$$= \frac{T.d/2}{\frac{\pi d^4}{64}}$$
$$\sigma_{max} = \frac{32T}{\pi d^3}$$

Deflection or wind – up angle:

Under the action of an axial torque the deflection of the spring becomes the "wind – up" angle of the spring which is the angle through which one end turns relative to the other. This will be equal to the total change of slope along the wire, according to area – moment theorem

$$\theta = \int_0^L \frac{M\,dL}{EI} \quad \text{but } M = T$$

$$= \int_0^L \frac{T.dL}{EI} = \frac{T}{EI}\int_0^L dL$$

Thus, as 'T' remains constant

$$\theta = \frac{T.L}{EI}$$

Futher

$$L = \pi D.n$$

$$I = \frac{\pi d^4}{64}$$

Therefore, on substitution, the value of θ obtained is

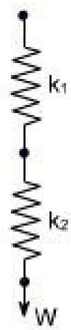
$$\boxed{\theta = \frac{64T.D.n}{E.d^4}}$$

Springs in Series: If two springs of different stiffness are joined endon and carry a common load W, they are said to be connected in series and the combined stiffness and deflection are given by the following equation.

$$\frac{W}{k} = x_1 + x_2 = \frac{W}{k_1} + \frac{W}{k_2}$$

or

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$



Springs in parallel: If the two springs are joined in such a way that they have a common deflection

„x"; then they are said to be connected in parallel. In this case the load carried is shared between the two springs and total load $W = W^1 + W^2$

Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. ***Solid length.*** When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid.*** The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$L_S = n'.d$

where n' = Total number of coils, and

d = of the wire.

2. ***Free length***. The free length of a compression spring, as shown in Fig. is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta_{max} + 0.15\ \delta_{max}$

The following relation may also be used to find the free length of the spring, *i.e.*

$L_F = n'.d + \delta_{max} + (n' - 1) \times 1$ mm

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. ***Spring index***. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. ***Spring rate***. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

5. ***Pitch***. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $p = \dfrac{\text{Free length}}{n' - 1}$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $p = \dfrac{L_F - L_S}{N'} + d$

where L = Free length of the spring,

Ls = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :

(a) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(b) The spring should not close up before the maximum service load is reached.

Note : In designing a tension spring (See Example 23.8), the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,

$$L_F = n.d + (n - 1)$$

and pitch of the coil, $$p = \frac{L_F}{n - 1}$$

Example 1. *Design a helical compression spring for a maxin deflection of 25 mm using the value of spring index as 5.*
The maximum permissible shear stress for spring wire is 420
84 kN/mm^2.
Take Wahl's factor, $K = \frac{4C-1}{} + \frac{0.615}{}$ *, where C =*

Solution. Given: W = 1000 N; δ = 25 mm; $C = D/d = 5$; τ = 420 MPa = 420 N/mm^2; G = 84 kN/mm^2 = 84 × 10^3 N/mm^2

1. Mean diameter of the spring coil

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \cdot 5-1}{4 \cdot 5-4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \cdot \frac{8\,W.C}{\pi d^2} = 1.31 \cdot \frac{8 \cdot 1000 \cdot 5}{\pi d^2} = \frac{16\,677}{d^2}$$

$\therefore$ d^2 = 16 677 / 420 = 39.7 or d = 6.3 mm

we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$D = C.d = 5\,d = 5 \times 6.401 = 32.005$ mm **Ans** ($\because$ $C = D/d = 5$)

and outer diameter of the spring coil,

$D_o = D + d = 32.005 + 6.401$ = 38.406 mm **Ans.**

2. Number of turns of the coils

Let n = Number of active turns of the coils.

we know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \cdot 1000(5)^3 n}{84 \cdot 10^3 \cdot 6.401} = 1.86\, n$$

∴ $n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns, n'

$= n + 2 = 14 + 2 = 16$ **Ans.**

3. *Free length of the spring*

We know that free length of the spring

$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$

$= 131.2$ mm **Ans.**

4. *Pitch of the coil*

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm } \textbf{Ans.}$$

Example 2. *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G =*

84 kN/mm².

Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, show-ing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa =

420 N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of

$W_2 = 2750$ N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring

$T = W_2 \cdot D/2 = 6875\, d$

We also know that twisting moment (T),

$6875\, d = (\pi/16) \cdot \tau \cdot d^3 = \pi/16 \cdot 420 \cdot d^3 = 82.48\, d^3$

∴ $d^2 = 6875 / 82.48 = 83.35$ or $d = 9.13$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm. ∴

Mean diameter of the spring coil,

$D = 5d = 5 \times 9.49 = 47.45$ mm **Ans.**

We know that outer diameter of the spring coil,

$D_o = D + d = 47.45 + 9.49 = 56.94$ mm **Ans.**

and inner diameter of the spring coil,

$D_i = D - d = 47.45 - 9.49 = 37.96$ mm **Ans.**

2. *Number of turns of the spring coil*

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for W = 500 N) is 6 mm.

We know that the deflection of the spring (δ),

$$= 8WD^3/Gd^4$$

∴ $n = 6 / 0.63 = 9.5$ say 10 **Ans.**