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Reynolds Number

A flow can be Laminar, Turbulent or Transitional in nature. This becomes a very important classification of flows and is brought out vividly by the experiment conducted by Osborne Reynolds (1842 - 1912). Into a flow through a glass tube as shown in Figure 1, he injected a dye to observe the nature of flow. When the speeds were small the flow seemed to follow a straight line path (with a slight blurring due to dye diffusion). As the flow speed was increased the dye fluctuates and one observes intermittent bursts. As the flow speed is further increased the dye is blurred and seems to fill the entire pipe. These are what we call Laminar, Transitional and Turbulent Flows.

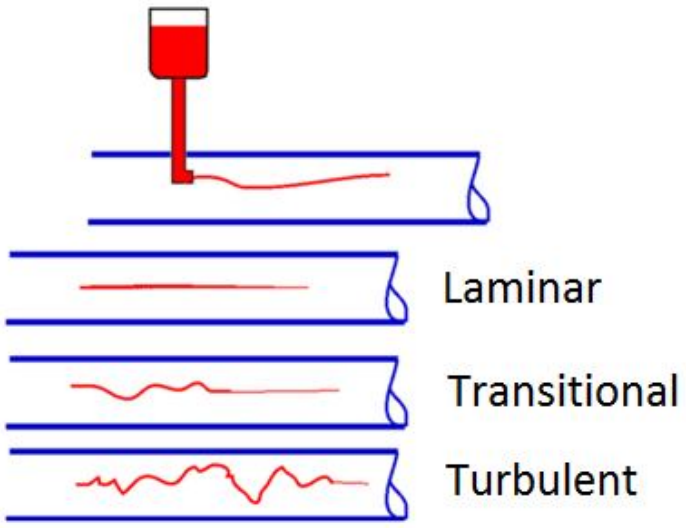

Figure 1: Reynolds Experiment

The kind of fluid flow is governed by the dimensionless number, called the Reynolds number. A flow in a pipe is laminar if the Reynolds Number (based on diameter of the pipe) is less than 2000 and is turbulent if it is greater than 4000. Transitional Flow prevails between these two limits.

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The Reynolds Number is given as:

$$R_e = \frac{\rho u d}{\mu}$$

Where

ρ=density,

μ= viscosity,

u = *mean velocity*, and

d=diameter.

- Laminar flow: Re< 2000
- Transitional flow: 2000 < Re < 4000
- Turbulent flow: Re > 4000

Laminar and Turbulent Flow

The phenomenon was first investigated in the 1880s by Osborne Reynolds in an experiment which has become a classic in fluid mechanics.

Laminar Flow

Laminar flow is a type of fluid flow in which the fluid travels smoothly or in regular paths. In laminar flow, sometimes called streamline flow, the velocity, pressure, and other flow properties at each point in the fluid remain constant. Laminar flow over a horizontal surface may be thought of as consisting of thin layers, or laminas, all parallel to each other. The fluid in contact with the horizontal surface is stationary, but all the other layers slide over each other.

Laminar flow in a straight pipe may be considered as the relative motion of a set of concentric cylinders of fluid, the outside one fixed at the pipe wall and the others moving at increasing speeds as the centre of the pipe is approached. Oil flow through a thin tube or blood flow through capillaries is laminar.

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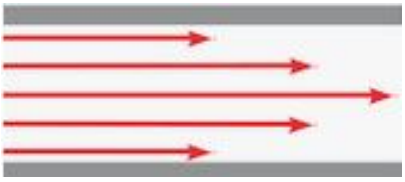

Laminar Flow

Salient Points

- The motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls.
- A laminar flow is one in which the fluid particles move in layers or lamina with one layer sliding over the other.
- Fluid element move in well defined paths and they retain the same relative position at successive cross-section of the flow passage.
- This flow is also known as the streamline flow.
- Laminar flow is common only in cases in which the flow channel is relatively small.
- The fluid is moving slowly.
- Viscosity is relatively high.

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Examples:

1. Blood flow throughout most of the circulatory
2. Flow of honey or thick syrup from a bottle.
3. A deck of new cards, as a rough analogy, may be made to "flow" laminarly.
4. Smoke rising in a straight path from a cigarette is undergoing laminar flow.

Turbulent Flow

Turbulent flow is a type of fluid flow in which the fluid undergoes irregular fluctuations or mixing. In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction. Turbulent flows are characterized by a mixing action throughout the flow field caused by eddies in the flow.

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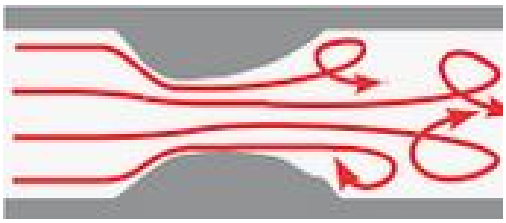
Thus in turbulent flow the mixing due to molecular motion is superimposed by mixing due to eddies. Though the mixing due to eddies may appear to be small but they have important consequences.



Turbulent Flow

Salient Points

- It is this type of flow in which the fluid particles move in a zig-zag way.
- The air or water swirls and eddies while its overall bulk moves along a specific direction.
- Most common type of flow.
- All fluid particles are disturbed and they mix with each other.
- Transfer of momentum to adjacent layer is continuous in turbulent flow.
- The high energy loss due to the formation of eddies in the flow.

Examples:

1. The blood flow in arteries.
2. Oil transport in pipelines.
3. The flow of wind and rivers even if the currents are gentle.
4. The flow through pumps and turbines.
5. Smoke from a cigarette after rising a small distance usually changes to turbulent flow, as it eddies and swirls from its regular path.

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Difference between Laminar and Turbulent Flow

S.no	Laminar Flow	Turbulent Flow
1.	It is a fluid flow in which the fluid layers move parallel to each other and do not cross each other.	It is a fluid flow in which the fluid layers cross each other and do not move parallel to each other.
2.	The laminar flow generally occurs in the fluid flowing with low velocity.	The turbulent flow occurs when the fluid flows with high velocity.
3.	Laminar flow occurs in the small diameter pipes in which fluid flows with low velocity.	Turbulent flow occurs in large diameter pipes in which fluid flows with high velocity.
4.	The fluid flow is laminar when the value of Reynolds number (Re) is less than 2000.	The fluid flow is turbulent when the value of Reynolds number is greater than 4000.
5.	Shear stress in laminar flow depends only on the viscosity of the fluid and independent of the density.	Shear stress in the turbulent flow depends upon the density of the fluid.
6.	The fluid flow is very orderly i.e. there is no mixing of adjacent layers of the fluid and they move parallel to each other and also with the walls of the pipe.	The fluid flow is not orderly i.e. there is mixing of adjacent layers of fluid with each other and they do not move parallel to each other and also with the walls of the pipe.

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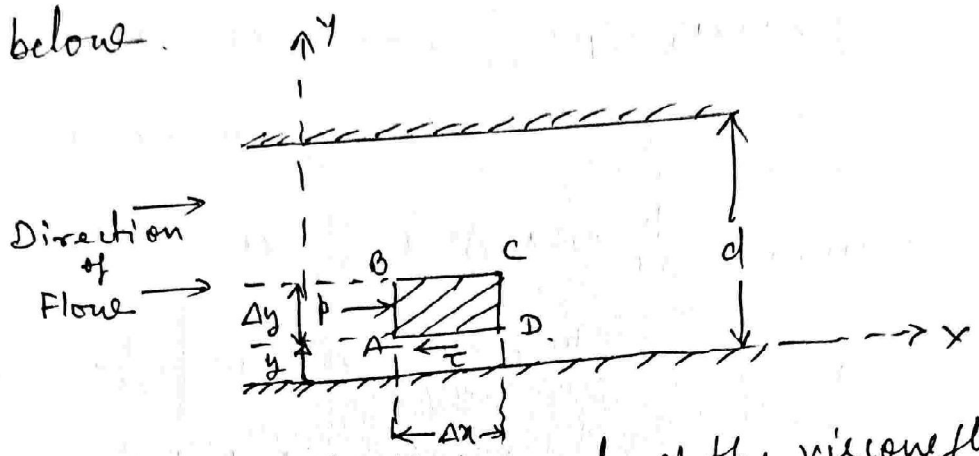
Viscous Flow through Parallel Plates

consider two parallel plates as shown below.



considering a fluid element of the viscous fluid of length = Δx,
height = Δy, and
width = 1 in the direction perpendicular to the paper.

Let $p \rightarrow$ pressure on face AB and
$\tau \rightarrow$ shear stress on face AD

Then,

Shear force on face AD = $\tau \times \Delta x \times 1$

face BC = $\left(\tau + \frac{\partial \tau}{\partial y} \times \Delta y\right) \times \Delta x \times 1$

pressure force on face AB = $p \times \Delta y \times 1$

face CD = $\left(p + \frac{\partial p}{\partial x} \times \Delta x\right) \times \Delta y \times 1$

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For steady and uniform flow.

Resultant force in the direction of flow is zero

Therefore

$$\Rightarrow (p \times \Delta y \times 1) - \left(p + \frac{\partial p}{\partial x} \times \Delta x\right) \times \Delta y \times 1 - \tau \times \Delta x \times 1 + \left(\tau + \frac{\partial \tau}{\partial y} \times \Delta y\right) \times \Delta x \times 1 = 0$$

$$\Rightarrow \text{OR}, \quad -\frac{\partial p}{\partial x} \times \Delta x \times \Delta y + \frac{\partial \tau}{\partial y} \times \Delta y \times \Delta x = 0.$$

$$\Rightarrow \text{OR}, \quad -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = 0 \quad \text{or} \quad \boxed{\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}}$$

Now, (a) the Shear stress distribution,
(b) Velocity distribution across a section:
(Ratio of maximum velocity to average velocity)
and (c) difference of pressure head for a given length
may be calculated.

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(i) Velocity Distribution.

Since $\tau = \mu \frac{du}{dy}$

$$\therefore \frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[\mu \cdot \frac{du}{dy}\right] = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\therefore \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating w.r.t. y,

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} \cdot y + C_1$$

Integrating again.

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x} \cdot \frac{y^2}{2} + C_1 y + C_2 \quad \text{——(i)}$$

C_1 and C_2 → constants of integration.

Using boundary conditions as

(i) At $y = 0$, $u = 0$ and (ii) at $y = d$, $u = 0$.

∴ From eqⁿ (1) on substitution

$$0 = 0 + C_1 \times 0 + C_2 \longrightarrow \boxed{C_2 = 0}$$

$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x} \times \frac{d^2}{2} + C_1 \times d + 0$$

$$\therefore \boxed{C_1 = -\frac{1}{\mu}\frac{\partial p}{\partial x} \times \frac{d}{2}}$$

$$\therefore u = \frac{1}{\mu}\frac{\partial p}{\partial x} \times \frac{y^2}{2} - \frac{d}{2\mu}\frac{\partial p}{\partial x} \times y$$

or $$\boxed{u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[dy - y^2\right]}$$

Since, μ, $\frac{\partial p}{\partial x}$ and d are constant, thus u varies with y^2, thus it is a equation of parabola. and velocity distribution is parabolic.

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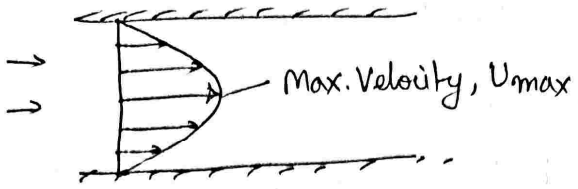

Velocity Distribution across a section of parallel plates

Ratio of Maximum Velocity to Average Velocity ($\frac{U_{max.}}{\bar{u}}$)

$U_{max.}$ when $y = \frac{d}{2}$

$$\therefore \quad U_{max.} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[d \times \frac{d}{2} - \left(\frac{d}{2}\right)^2\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{d^2}{2} - \frac{d^2}{4}\right] = \frac{-1}{2\mu}\frac{\partial p}{\partial x} \times \frac{d^2}{4}$$

$$\text{or } U_{max.} = -\frac{1}{8\mu}\frac{\partial p}{\partial x} \times d^2$$

Average Velocity $\bar{u} = \frac{Q}{\text{Area.}} = \frac{Q}{d \times 1}$

$$Q = \int_0^d dQ = \int_0^d -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[dy - y^2\right] \times \left[dy \times 1\right]$$

$[\because Q = V \times \text{Area}]$

$[dQ = \text{Flow through strip}]$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{dy^2}{2} - \frac{y^3}{3}\right]_0^d$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{d^3}{6} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}d^3$$

$$\therefore \quad \bar{u} = \frac{Q}{\text{Area}} = \frac{-\frac{1}{12\mu}\frac{\partial p}{\partial x} \cdot d3}{d \times 1}$$

$[\text{Area} = d \times 1]$

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x} \times d^2 \quad \text{or} \quad \frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{d2}$$

$$\therefore \quad \frac{U_{max.}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x} \times d^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x} \times d^2} = \frac{12}{8} = \frac{3}{2}$$

$$\frac{U_{max.}}{\bar{u}} = \frac{3}{2}$$

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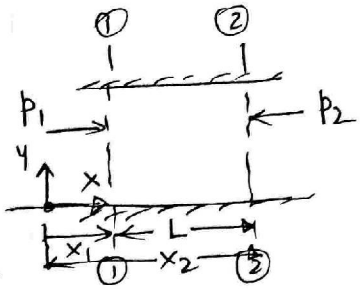
Drop of Pressure head for a given length

Since $\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x} \times d^2$

$$\therefore \quad \frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{d^2}$$



$$\therefore \quad \int_2^1 dp = \int_2^1 -\frac{12\mu\bar{u}}{d^2}\,dx$$

$$\text{or} \quad p_1 - p_2 = -\frac{12\mu\bar{u}}{d^2}[x_1 - x_2] = \frac{12\mu\bar{u}}{d^2}[x_2 - x_1]$$

$$\text{or,} \quad p_1 - p_2 = \frac{12\mu\bar{u}L}{d^2} \qquad [\text{Since } x_1 - x_2 = L]$$

Drop of pressure head, h_f

$$\boxed{h_f = \frac{p_1 - p_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g d^2}}$$

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(iii) Shear Stress Distribution.

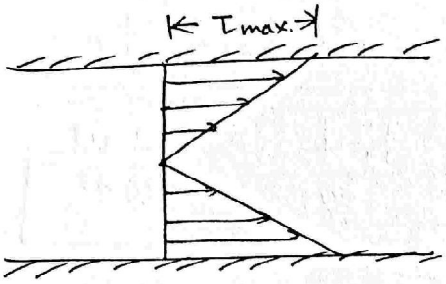
$$\tau = \mu \frac{\partial u}{\partial y}$$

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(dy - y^2)\right]$$

$$= \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(d - 2y)\right]$$

$$\boxed{\tau = -\frac{1}{2}\frac{\partial p}{\partial x}[d - 2y]}$$

Since $\frac{\partial p}{\partial x}$ and t are constant. Hence, Shear stress τ varies linearly with y.



Shear stress distribution across a section of parallel plates.

When $y = 0$, shear stress is Maximum, and when $y = \frac{d}{2}$, shear stress is zero

Thus, Maximum Shear stress, $\boxed{\tau_{max.} = -\frac{1}{2}\frac{\partial p}{\partial x} \times d.}$

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Prob.1 Water at 15°c flows between two large parallel plates at a distance of 1.6 mm apart. Determine
(i) The maximum velocity
(ii) the pressure drop per unit length, and
(iii) the shear stress at the walls of the plates if the average velocity is 0.2 m/s.
The viscosity of water at 15°c is given as 0.01 poise.

Sol. **Given.** $d = 1.6\,mm = 1.6\times10^{-3}\,m.$

$\bar{u} = 0.2\ m/s.$

$\mu = 0.01\ poise = 0.001\ \frac{N\text{-}s}{m^2}$

(i) $U_{max.} = \frac{3}{2}\bar{u} = \frac{3}{2}\times 0.2 = 0.3\ m/s$ – Ans

(ii) $p_1 - p_2 = \frac{12\mu\bar{u}L}{d^2}$

$= 12\times\frac{0.001\times0.2\times1}{(1.6\times10^{-3})^2}$ $\left[\because L = \frac{1}{m.}\right]$

or $\frac{\partial p}{\partial x} = 937.44\ \frac{N}{m^2}$ per m. length – Ans

(iii) $\tau_{wall} = \tau_{max.} = -\frac{1}{2}\frac{\partial p}{\partial x}\times d$

$= -\frac{1}{2}\times 937.44\times 1.6\times10^{-3}$

$= 0.74995\ \frac{N}{m^2}$ – Ans

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Prob.2 Two fixed parallel plates kept 8 cm apart have laminar flow of oil between them with a maximum velocity of 1.5 m/s. Taking dynamic viscosity of oil to be $\mu = 2.0$ N-s/m^2. Calculate:
(i) the shear stress at the plates.
(ii) the discharge per metre width.
(iii) the pressure difference between two points 25 m apart.
(iv) velocity at 2 cm from the plate.
(v) the velocity gradient at the plates end.

Sol. Given. $d = 8\text{ cm} = 0.08\text{ m}$.

$U_{max.} = 1.5$ m/s

$\mu = 2.0 \frac{\text{N-s}}{\text{m}^2}$

$L = 25$ m.

(i) $\tau_{\text{plates wall}} = \tau_{max.} = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right) \times d$

Since, $\frac{\partial p}{\partial x} = \frac{-12\mu\bar{u}}{d^2}$

$= -\frac{12\mu}{d^2} \times \left[\frac{2}{3} \times U_{max.}\right]$ $\left[\because \frac{U_{max.}}{\bar{u}} = \frac{3}{2}\right]$

$= \frac{-12 \times 2.0}{(0.08)^2} \times \frac{2}{3} \times 1.5$

$= -3750 \frac{N}{m^2}$ per m.

$\therefore \tau_{max.} = -\frac{1}{2} \times (-3750) \times 0.08$

$= 150\ \text{N/m}^2$ — Ans

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(ii) $Q = \bar{u} \times \text{Area} = \bar{u} \times d \times 1$

$= 0.08 \times \left(\frac{2}{3} \times 1.5\right) \times 1$ $\quad \left[\bar{u} = \frac{2}{3} u_{max.} = \frac{2}{3} \times 1.5 = 1 \text{ m/s}\right]$

$= 0.08 \text{ m}^3/\text{s}$. — Ans

(iii) $p_1 - p_2 = \frac{12 \mu \bar{u} \times L}{d^2}$

$= \frac{12 \times 2.0 \times 1.0 \times 25}{(0.08)^2}$

$= 93750 \text{ N/m}^2$ — Ans

(iv) $u = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(dxy - y^2)$ at $y = 2\text{cm} = 0.02\text{m}$.

$= \frac{1}{2 \times 2.0} \times 3750 \times \{0.08 \times 0.02 - (0.02)^2\}$

$= 1.125 \text{ m/s}$. — Ans

(v) Velocity gradient, $\left(\frac{du}{dy}\right)$ at surface of plate i.e $y = 0$.

$\frac{du}{dy} = \frac{d}{dy}\left[\frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(dxy - y^2)\right]$

$= \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(d - 2y)$

$\therefore \frac{du}{dy} = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right) \times d$ at $y = 0$.

$= \frac{1}{2 \times 2.0} \times (3750) \times 0.08$

$= 75 \text{ m/s per m}$. — Ans

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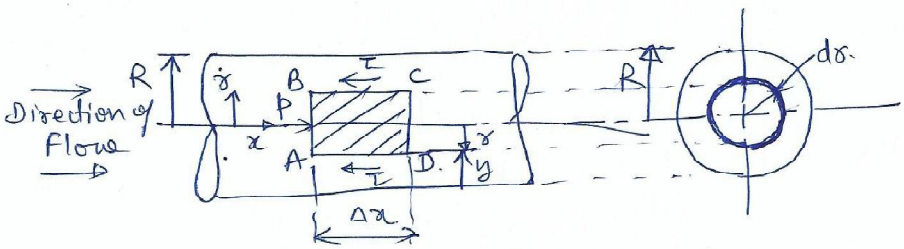
Viscous Flow through Circular Pipe



For the viscous or laminar flow.

$$Re = \frac{\rho V D}{\mu} < 2000.$$

Considering a pipe of radius R. A fluid element of radius r sliding in a fluid element of radius (r+dr).

Then,

pressure force on face AB = $p \times \pi r^2$

face CD = $\left(p + \frac{\partial p}{\partial x} \times \Delta x\right) \times \pi r^2$

Shear force on Surface of the element

$= \tau \times 2\pi r \times \Delta x$

Since, there is no acceleration, sum of all the forces in the direction of flow must be zero

$$\therefore \quad p \times \pi r^2 - \left(p + \frac{\partial p}{\partial x} \times \Delta x\right) \times \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or, $$-\frac{\partial p}{\partial x} \times \Delta x \times \pi r^2 = \tau \times 2\pi r \times \Delta x$$

$$-\frac{\partial p}{\partial x} \cdot r = 2\tau$$

or,

$$\boxed{\tau = -\frac{\partial p}{\partial x} \times \frac{r}{2}}$$

Thus, shear stress distribution is linear.

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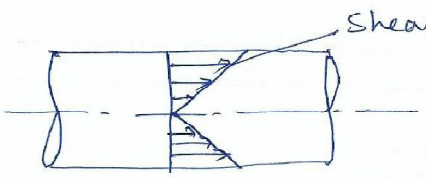


Shear Stress Distribution across a section.

Velocity Distribution.

Since, $\tau = \mu \cdot \frac{du}{dy}$

$y = R - r$ (As y is measured from wall of pipe)

$\therefore dy = -dr$

$\Rightarrow \therefore \quad \tau = \mu \cdot \frac{du}{-dr} = -\mu \frac{du}{dr}$

$\Rightarrow \quad -\mu \cdot \frac{du}{dr} = -\left(\frac{\partial p}{\partial x}\right) \times \frac{r}{2}$

$\Rightarrow$ or. $\quad \frac{du}{dr} = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) \times r$

Integrating w.r.t. r.

$$u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \times r^2 + C$$

Using Boundary condition

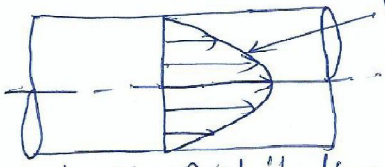
at $r = R$, $u = 0$.

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \times R^2 + C$$

$$\therefore \quad C = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \times R^2$$

$\therefore$

$$\boxed{u = \frac{-1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right]}$$ → Equation of parabola



Thus, the velocity distribution across a section of a pipe is parabolic.

Velocity Distribution across a section

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Ratio of Max. velocity to average velocity

$$\left(\frac{U_{max.}}{\bar{u}}\right)$$

$U_{max.}$ is at $r = 0$.

$$\therefore \boxed{U_{max.} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} \times R^2}$$

Since $\bar{u} = \frac{Q}{\text{area}}$

where $Q = \int_0^R \delta Q = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r \times dr.$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4}$$

[δQ = flow through a circular ring of radius r and thickness dr]

$$= \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4$$

$$\therefore \quad \bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4}{\pi R^2}$$

or, $$\boxed{\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2}$$

$$\frac{U_{max.}}{\bar{u}} = \frac{-\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \times R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) \times R^2} = 2.0$$

$$\therefore \quad \boxed{U_{max.} = 2.0 \times \bar{u}}$$

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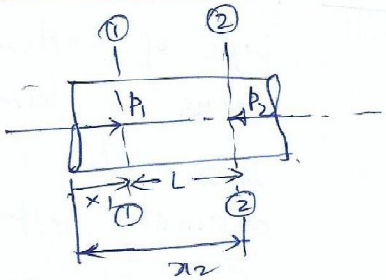
Pressure drop for a given length



Since, $\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) \times R^2$

or $-\frac{\partial p}{\partial x} = \frac{8\mu\bar{u}}{R^2}$

$\therefore \quad -\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\, dx$

$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2]$

or $p_1 - p_2 = -\frac{8\mu\bar{u}}{R^2}(x_1 - x_2)$

$= \frac{8\mu\bar{u}}{R^2} \times L \qquad [\because L = -(x_1 - x_2) = x_2 - x_1]$

$= \frac{8\mu\bar{u}L}{(D/2)^2}$

or, $p_1 - p_2 = \frac{32\mu\bar{u}L}{D^2}$

Loss of head. $h_f = \frac{p_1 - p_2}{\rho g}$

$$\boxed{\frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2}}$$

This formula is known as
Hagen Poiseuille Formula

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Prob.1 Oil of specific gravity 0.82 is pumped through a horizontal pipe line 15 cm in. diameter and 3 km long at the rate of 900 litres per minute. This pump has an efficiency of 68% and required 7.35 kW. to pump the oil. Determine the dynamic viscosity of oil and verify whether the flow is laminar

Sol. Given. $S = 0.82$ $D = 15\ cm = 0.15\ m.$

$L = 3\ km = 3000\ m.$ $Q = 900\ ltrs/min. = \frac{900\times10^{-3}}{60}\ m^3/s.$

$\eta_p = 68\ \%$

$P = 7.35\ kW.$

$$\bar{u} = \frac{Q}{A} = \frac{900\times10^{-3}}{60\times\frac{\pi}{4}\times(0.15)^2} = 0.84925\ m/s$$

$\Rightarrow$ Power required to pump oil $= \frac{w\times Q\times h_f}{\eta_p}$

$$\Rightarrow 7.35\times1000 = \frac{(0.82\times1000\times9.81)\times\frac{900\times10^{-3}}{60}\times h_f}{0.68}$$

$$\Rightarrow h_f = 41.42\ m.\ of\ oil.$$

$$\Rightarrow h_f = \frac{p_1-p_2}{w} = \frac{32\mu\bar{u}L}{w\times d^2}$$

$$\Rightarrow \therefore \mu = \frac{w d^2 h_f}{32\bar{u}L} = \frac{(0.82\times9810)\times(0.15)^2\times41.42}{32\times0.849\times3\times1000}$$

$$= 0.092\ Ns/m^2$$

$$Re = \frac{\rho\bar{u}d}{\mu} = \frac{(0.82\times1000)\times0.849\times0.15}{0.092}$$

$$= 1135$$

This is less than 2000, thus, the flow is laminar.

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Prob 2 At what distance r from the centre of a pipe of radius R does the average velocity occur in laminar flow?

Sol. Since,

$$u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2) \text{ and}$$

$$\bar{u} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right) \times R^2$$

when $u = \bar{u}$

$$\Rightarrow \quad -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2) = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right) \times R^2$$

$$R^2 - r^2 = \frac{1}{2} \times R^2$$

$\Rightarrow$ or,

$$\frac{1}{2}R^2 = r^2$$

$\Rightarrow$ or,

$$r = \frac{R}{\sqrt{2}}$$

$\Rightarrow$ or,

$$\boxed{r = 0.707R} \quad \text{— Ans}$$

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NAVIER-STOKES EQUATION

The Navier-Stokes equation is the most important equation in fluid mechanics. It is simply **f=ma** for a viscous fluid.

These are as given below for a Newtonian fluid such as water or air:

$$\mu \frac{\partial^2 \mathbf{v}}{\partial x^2} + \mu \frac{\partial^2 \mathbf{v}}{\partial y^2} + \mu \frac{\partial^2 \mathbf{v}}{\partial z^2} - \nabla p = \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v}$$

In vector notation, this can be written as:

$$\mu(\nabla \cdot \nabla)\mathbf{v} - \nabla p = \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v}$$

This is abbreviated to:

$$\mu \nabla^2 \mathbf{v} - \nabla p = \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v}$$

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This is the Navier-Stokes equation. It is f = ma written for a viscous fluid. It is usually written the other way round and divided by:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \mathbf{v}$$

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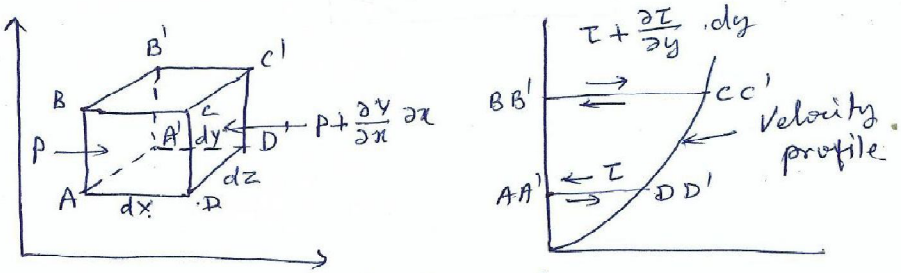
Relationship between Shear Stress and Pressure gradient



Considering any fluid element of
length $-dx$
thickness $-dy$ and
width $-dz$

Shearing force on the element as in above fig.

$$= \left[\tau + \left(\frac{\partial \tau}{\partial y}\right) \times dy \right] dx\,dz - \tau \cdot dx \cdot dz$$

$$= \frac{\partial \tau}{\partial y} \cdot dx\,dy \cdot dz$$

If p is the intensity of pressure.

pressure force on the element as in above fig.

$$= p \cdot dy \cdot dz - \left[p + \left(\frac{\partial p}{\partial x}\right) \times dx \right] dy \cdot dz$$

$$= -\frac{\partial p}{\partial x}\, dx\,dy \cdot dz$$

For equilibrium in steady flow, the acceleration is zero. The sum of pressure and visious forces in the x-direction must be zero.

$$\therefore \quad \frac{\partial \tau}{\partial y} \cdot dx \cdot dy \cdot dz - \frac{\partial p}{\partial x} \cdot dx \cdot dy \cdot dz = 0$$

For the concentration on pressure force only.

$$\frac{dp}{dx} = \frac{d\tau}{dy}.$$

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Since $\tau = \mu \frac{du}{dy}$

$$\therefore \quad \boxed{\frac{dp}{dx} = \mu \frac{d^2u}{dy^2}}$$

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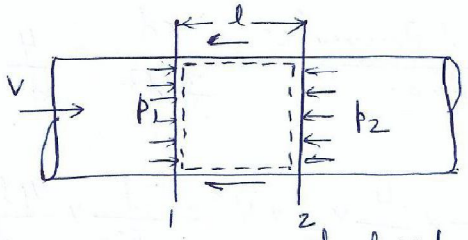
Head Loss in Turbulent Flow (DARCY EQUATION)



Force on control volume

Propelling force on the flowing fluid between the two sections.

$$= (p_1 - p_2) A.$$

As Viscous friction associated with fluid is proportional to
- pipe length, l,
- wetted perimeter, p and
- V^n [$V \rightarrow$ average flow velocity and
$n \rightarrow$ index depending upon material and nature of the pipe surface]

Thus, frictional resistance force can be written as

$$= f' p l V^2$$ [$n = 2$ with the turbulent flow in commercial pipes]

f' – non-dimensional factor whose value depends upon the material and nature of the pipe surface

Under equilibrium conditions,

$$(p_1 - p_2) A = f' p l V^2$$

or $$\frac{p_1 - p_2}{\omega} = \frac{f'}{\omega} \frac{p}{A} l V^2$$

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or $h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times l \times V^2$

$$\frac{P}{A} = \frac{\text{Welted Perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$$

$$\therefore \quad h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times l \times V^2 = \frac{f'}{\rho g} \times \frac{4 l V^2}{d}$$

Putting $\frac{f'}{\rho} = \frac{f}{2}$ where f is known as coefficient of friction.

$$\therefore \quad \boxed{h_f = \frac{4 \cdot f l V^2}{2 g \times d}}$$

Equation as above is known as Darcy - Weisbach equation

Sometimes, this equation is written as

$$\boxed{h_f = \frac{f \cdot l V^2}{2 g d}}$$

Then f is known as friction factor

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Empirical correlations for coefficient of friction.

(i) $4f = \frac{64}{Re}$ for laminar flow

(ii) $20 \times 10^3 < Re < 80 \times 10^3$

$$4f = \frac{0.3164}{(Re)^{0.25}}$$ (Blasius equation)

(iii) $4 \times 10^3 < Re < 3.2 \times 10^6$ (Nikuradse equation)

$$4f = 0.0032 + \frac{0.221}{(Re)^{0.237}}$$

(iv) $2 \times 10^4 < Re < 2 \times 10^6$

$$4f = 0.005 + \frac{0.396}{(Re)^{0.3}}$$ (Schiller equation)

Prob.1 An oil of specific gravity 0.85 and viscosity 0.05 poise flows through a 20cm diameter pipe at the rate of 75 litres per second. Find the head lost due to friction for a 500 m. length of pipe. Also calculate the power required to maintain this flow.

Sol. Given. $s = 0.85$ $\mu = 0.05$ poise
$d = 20$ cm. $Q = 75$ litres/s.
$h_f = ?$ $l = 500$ m., $P = ?$

$$V = \frac{Q}{A} = \frac{75 \times 10^{-3}}{\frac{\pi}{4} \times (0.2)^2} = 2.38 \text{ m/s.}$$

$$\mu = 0.05 \text{ poise} = 0.05 \times 0.1 = 0.005 \frac{Ns}{m^2}$$

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$$Re = \frac{\rho V d}{\mu}$$

$$= \frac{2.38 \times 0.2 \times 850}{0.005} = 81192$$

For this turbulent flow, using the ~~relation~~ Blasius relation for friction coefficient

$$4f = \frac{0.3164}{(Re)^{0.25}} = 0.0187$$

$$h_f = \frac{4 f l V^2}{2 g d}$$

$$= \frac{0.0187 \times 500 \times (2.38)^2}{2 \times 9.81 \times 0.2}$$

$$= 13.62 \text{ m of oil.}$$

$$\text{Power} = w Q h_f = \rho g Q h_f$$

$$= (850 \times 9.81) \times (75 \times 10^{-3}) \times 13.62$$

$$= 8505 \text{ watt} - \text{Ans}$$

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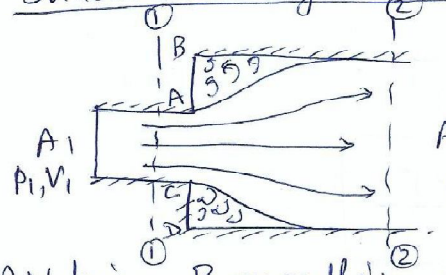
Minor Head Loses

(1) Sudden enlargement



Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{head loss due to sudden enlargement}$$

$z_1 = z_2$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

or $$h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \text{—— (i)}$$

From momentum consideration

$$p_1 A_1 + p_x (A_2 - A_1) - p_2 A_2 = \frac{w Q}{g}(V_2 - V_1)$$

The velocity at the annular wall AB and CD is very small and so the pressure there can be assumed to be equal to the pressure of incoming fluid i.e. $p_x = p_1$

$$\therefore \quad (p_1 - p_2) A_2 = \frac{w Q}{g}(V_2 - V_1)$$

or $$\frac{p_1 - p_2}{w} = \frac{Q}{A_2 g}(V_2 - V_1)$$

$$= \frac{V_2 (V_2 - V_1)}{g}$$

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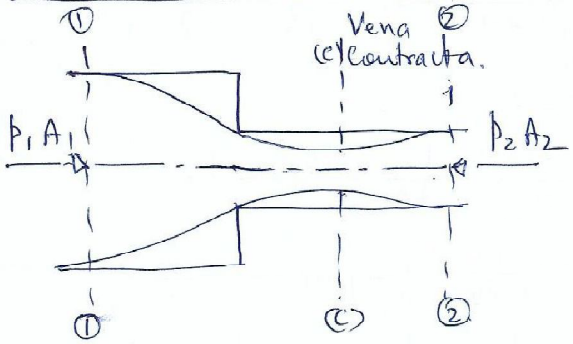
Substituting in Bernoulli's equation (i)

$$h_e = \frac{V_2(V_2 - V_1)}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \left(\frac{V_1 - V_2}{2g}\right)^2$$

$$\therefore \boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

(ii) Sudden contraction



Head loss due to sudden contraction
= actual head loss due to enlargement from section c-c to section 2-2.

$$= \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2$$

From continuity equation.

$$A_c V_c = A_2 V_2 \text{ or } \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{A_c/A_2} = \frac{1}{C_c} \left[\because C_c = \frac{A_c}{A_2}\right]$$

$$\therefore h_e = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$= \frac{K V_2^2}{2g}, \text{ where } K = \left[\frac{1}{C_c} - 1\right]^2$$

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If C_c is assumed to be $= 0.62$

$$K = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

$$\therefore \quad h_c = \frac{KV_2^2}{2g} = 0.375\frac{V_2^2}{2g}$$

If C_c is not given then,

$$h_c = 0.5\frac{V_2^2}{2g}$$

(iii) Head loss at the entrance of a pipe

$$h_i = 0.5\frac{V^2}{2g}$$

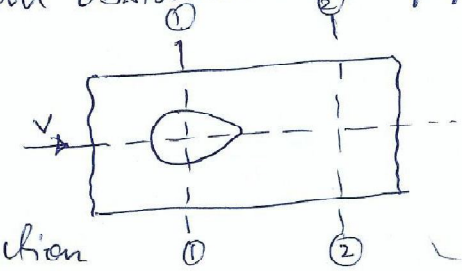
$V \rightarrow$ Velocity of liquid in pipe

(iv) Head loss at the exit of pipe.

$$h_o = \frac{V^2}{2g}$$

$V \rightarrow$ Velocity at outlet of pipe

(v) Loss of head due to an obstruction in a pipe



$$= \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)$$

where

a – Max. area of obstruction
A – Area of pipe
V – Velocity of liquid in pipe.

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Hydraulic Gradient and Total Energy Line

Hydraulic Gradient Line (H.G.L.)

→ A line which gives the sum of pressure head and the datum head.

i.e. $\left(\frac{p}{w}\right) + z$.

Total Energy Line (T.E.L.)

→ The line which gives the sum of pressure head, datum head and Kinetic head

i.e. $\frac{p}{w} + z + \frac{v^2}{2g}$.

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Prob2 The rate of flow of water pumped into a pipe ABC, which is 200 m long, is 20 litres/s. The pipe is laid on an upward slop of 1 in 40. The length of the portion AB is 100 m and its diameter is 100 mm, while the length of the portion BC is also 100 m but its diameter is 200 mm. The change of diameter at B is sudden. The flow is taking place from A to C, where the pressure at A is 19.62 N/cm^2 and end C is connected to a tank. Find the pressure at C and draw the hydraulic gradient and total energy line. Take $f = 0.008$.

Sol. Given. $L = 200$ m., $Q = 20$ litres/s $= 0.02$ m^3/s.

Slope $i = 1/40$, $l_{AB} = 100$ m, $d_{AB} = 0.1$ m.

$l_{BC} = 100$ m, $d_{BC} = 0.2$ m.

$p_A = 19.62$ N/cm^2 $= 19.62 \times 10^4$ N/cm^2

$f = 0.008$.

$$V_A = \frac{Q}{A_{AB}} = \frac{0.02}{\frac{\pi}{4} \times (0.1)^2} = 2.54 \text{ m/s}.$$

$$V_2 = \frac{Q}{A_{BC}} = \frac{0.02}{\frac{\pi}{4} \times (0.2)^2} = 0.63 \text{ m/s}$$

From Bernoulli's equation

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\rho g} + \frac{V_C^2}{2g} + z_C + h_{AC}$$

$h_{AC} = h_{AB} + h_{BC} + h_{enlargement}$ $(h_{AB} + h_{BC} + h_e)$

$$h_{AB} = \frac{4 f l_{AB} V_A^2}{2gd} = \frac{4 \times 0.008 \times 100 \times (2.54)^2}{2 \times 9.81 \times 0.1} = 10.52 \text{ m}$$

$$h_{BC} = \frac{4 f l_{BC} V_C^2}{2gd} = \frac{4 \times 0.008 \times 100 \times (0.63)^2}{2 \times 9.81 \times 0.2} = 0.323 \text{ m}.$$

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$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(2.54 - 0.63)^2}{2 \times 9.81} = 0.186 \text{ m.}$$

$$h_{AC} = \underset{(AB)}{h_{f1}} + h_e + \underset{(BC)}{h_{f2}}$$

$$= 10.52 + 0.186 + 0.323 = 11.03 \text{ m.}$$

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\rho g} + \frac{V_C^2}{2g} + z_C + 11.03.$$

$z_A = 0.$

$$z_C = \frac{1}{40} \times L = \frac{1}{40} \times 200 = 5 \text{ m.}$$

$p_A = 19.62 \times 10^4 \text{ N/m}^2$

$V_A = V_1 = 2.54 \text{ m/s}, \quad V_2 = V_3 = 0.63 \text{ m/s.}$

$$\therefore \frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{(2.54)^2}{2 \times 9.81} + 0 = \frac{p_C}{\rho g} + \frac{(0.63)^2}{2 \times 9.81} + 11.03$$

$$\therefore \frac{p_C}{\rho g} = 4.278 \text{ m.}$$

$p_C = 4.278 \times 1000 \times 9.81 \text{ N/m}^2$

$= 4.196 \text{ N/cm}^2 \text{ — Ans}$

$p_C = 4.196 \text{ N/cm}^2$

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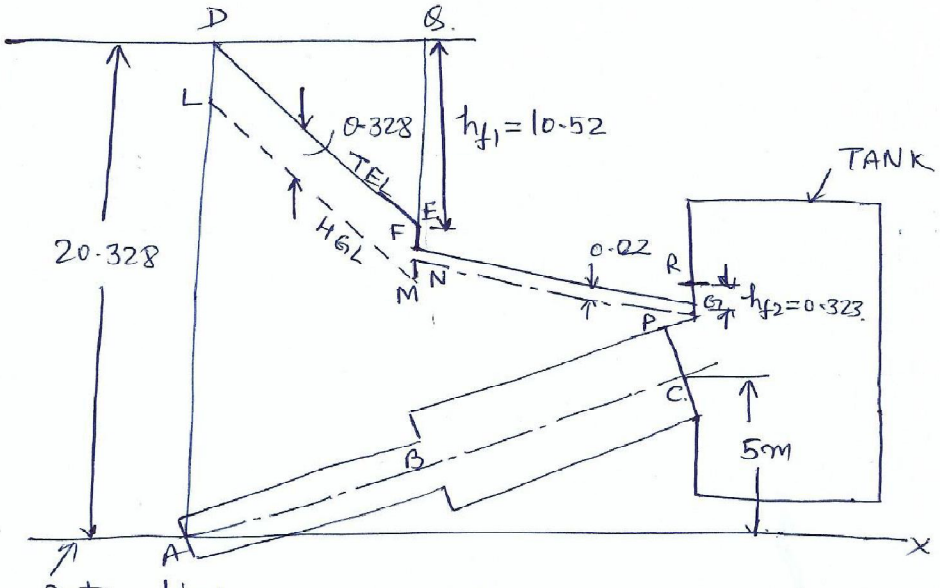

TEL

Total Energy at $A = \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{19·62 \times 10^4}{1000 \times 9·81} + \frac{(2·54)^2}{2 \times 9·81} + 0 = 20·328 m$

" at $B = 20·328 - h_{f1} = 20·328 - 10·52 = 9·808$ m.

$EF = h_e = 0·186$.

TEL $\longrightarrow$ DEFG.

HGL

$DL = \frac{V_A^2}{2g} = h_e = 0·328$

$MN = \frac{V_c^2}{2g} = 0·02$

HGL $\longrightarrow$ LMNP

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Pipe Network

A pipe network is an interconnected system of pipes forming several loops or circuits. The pipe networks have been used in municipal water distribution system and many other supply systems. The necessary conditions for the networks of pipes are.

(i) The flow into each junction must be equal to the flow out of the junction.

(ii) The algebraic sum of head loses round each loop must be zero.

(iii) The head loss in each pipe is expressed as $h_f = rQ^n$. The value of r depends upon the length of pipe, diameter of pipe and co-efficient of friction of pipe. The value of n for turbulent flow is 2.

$$h_f = \frac{4flV^2}{2gd} = \frac{4fl\left(\frac{Q}{A}\right)^2}{2gd} = \frac{4flQ^2}{2gd\times\left(\frac{\pi}{4}d^2\right)^2}$$

$$= \frac{4flQ^2}{2gd\times\left(\frac{\pi}{4}\right)^2\times d^4}$$

$$\boxed{h_f = rQ^2} \quad \left[\text{where } r = \frac{4fl}{2g\times\frac{\pi}{4}\times d^5}\right]$$